Knowledge Creation and Diffusion with Limited Appropriation

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Introduction

- Questions and motivation:
  - How do intellectual property rights affect growth?
  - How strong should they be?
- Literature on diffusion of knowledge as a motor for growth
- Consider the incentives to innovate v.s. imitate
  - Free riding on others' innovation reduces incentives to innovate
  - When learning has a cost, holdup from innovators
- Allocation of property rights affects both margins

Model

- Two additional modeling features
  1. Nash bargaining/patent strength for surplus division
  2. Congestion in knowledge diffusion
- Long run growth & Optimal property rights in a calibrated model

Related literature

- Knowledge diffusion and growth
  - Kortum 1997, Alvarez Buera Lucas 2008, Lucas 2009, Lucas Moll 2014, Perla and Tonetti 2014, Luttmer 2007, Buera Oberfield 2020
  - Luttmer 2015, Konig Lorenz Zilibotti 2016, Benhabib Perla Tonetti 2019, Jovanovic Wang 2023
- Patents and innovation
  - Boldrin Levine 2008, Acemoglu Akcigit 2012
  - Akcigit Celik Greenwood 2016
- Quantification and empirical
  - Garcia-Macia Hsieh Klenow 2019
  - Bloom Schankerman Van Reenen 2013, Kankanhalli Kwan 2022

Outline

1. An illustrative model
2. Dynamic model - Endogenous growth framework
3. Calibration
4. Long run growth
5. Optimal policies: Ramsey problem

An illustrative model

An illustrative model

- Endowment economy
- A unit mass of firms $z \sim F(z)$
- Can improve productive capacity through innovation or imitation
- Two stages:
  1. Innovation: firms decide whether to innovate
  2. Diffusion: innovators and imitators meet for knowledge transfers

Innovation and diffusion

- Innovation
  - Creates productive capacity μz at a convex cost $c(\mu) z$
  - α fraction innovate

- Diffusion
  - Innovators and imitators match in the technology transfer market
    - Matching function $M(\alpha, 1-\alpha)$. CRS
    - Canonical case: Cobb-Douglas $M(\alpha, 1-\alpha) = \alpha^{\omega}(1-\alpha)^{1-\omega}$
    - Contact rate for innovators: $\lambda(\alpha) = M(\alpha, 1-\alpha)/\alpha$
    - Contact rate for imitators: $\iota(\alpha) = M(\alpha, 1-\alpha)/(1-\alpha)$
  - Bargain over surplus $S = \mu z$
    - Innovators' bargaining weight $\beta \in [0, 1]$

Bargaining weight and IP

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- An enforcement game:
  - Innovator makes a take-it-or-leave-it offer t, e.g., license fee
  - If imitator rejects and copy unlicensed, probability $\beta \in [0, 1]$ get caught
  - If caught, cannot use knowledge (output zero)

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  - If caught, cannot use knowledge (output zero)
- IC constraint:

$$S - t \geq (1 - \beta)\, S$$

- So, transfer fee:

$$t = \beta S = \beta \mu z$$

The decentralized economy

- Innovators' problem

$$\max_{\mu} \; \left[\left(1+\lambda\left(\alpha\right)\beta\right)\mu - c\left(\mu\right) \right] z$$

Optimality condition for innovation

$$c'\left(\mu\right) = 1+\lambda\left(\alpha\right)\beta \tag{1}$$

The decentralized economy

- Innovators' problem

$$\max_{\mu} \; [(1+\lambda(\alpha)\beta)\mu - c(\mu)]\, z$$

Optimality condition for innovation

$$c'(\mu) = 1 + \lambda(\alpha)\beta \quad (1)$$

- Marginal firm $\underline{z}$ indifferent between innovating and learning:

$$[(1+\lambda(\alpha)\beta)\mu - c(\mu)]\, \underline{z} = \iota(\alpha)(1-\beta)\mu\mathbb{E}[z|\, z \geq \underline{z}] \quad (2)$$

- Innovator share $\alpha = 1 - F(\underline{z})$

Optimal Innovator Appropriation

- Planner:

$$Y = \max_{\alpha,\mu} \underbrace{\int_{\underline{z}} (\mu - c(\mu))\, z dF(z)}_{\text{output from incumbents}} + \underbrace{M(\alpha, 1-\alpha)\, \mathbb{E}\left[z \middle| z \geq \underline{z}\right] \mu}_{\text{surplus from matching}} \quad \text{s.t. (1) and (2)}$$

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- Optimal bargaining weight:

$$\frac{dY}{d\alpha} = \underbrace{(\lambda(\alpha) - \iota(\alpha))\,(\omega - \beta)\, \mathbb{E}[z]\, \mu}_{\text{Congestion: Hosios}}$$

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& - \underbrace{(1-\beta)\,\lambda(\alpha)\,(\mathbb{E}[z] - \underline{z})\,\mu}_{\text{Displacement externality}} \\
& + \underbrace{(1-\beta)\,M\mathbb{E}[z]}_{\text{Innovation spillover}} \frac{\partial \mu}{\partial \alpha}
\end{aligned}$$

Heterogeneity: a Pareto distribution example

- Knowledge distribution: $F(z) = 1 - \left(\frac{\underline{z}}{z}\right)^{\zeta}$, where $\zeta > 1$
- Matching function: $M(\alpha, 1-\alpha) = \kappa\alpha^{\omega}(1-\alpha)^{1-\omega}$, with $\kappa = 0.5$ and $\omega = 0.5$
- Innovation cost function: $c(\mu) = \frac{\chi}{1+\frac{1}{\varepsilon}}\mu^{1+\frac{1}{\varepsilon}}$, with $\chi = 2$ and $\varepsilon = 1$

Pareto	Mean/Margin	Optimal innovator appropriation β			Innovator	% Output
ζ	$\frac{\mathbb{E}[z]}{\underline{z}}$	Hosios	− Displacement	+ Spillover	share α	diffusion
20	1.05	0.5	0.5	0.5	82%	35%
2	2	0.5	0.28	0.36	47%	57%
1.1	11	0.5	0	0.13	1%	92%

Dynamic Model

Environment

- Time is continuous and infinite, $t \in [0, \infty)$
- Representative household supplies labor L and has preference

$$\int_0^\infty e^{-\rho t} \log\left(C\left(t\right)\right) dt$$

- Final good is a CES aggregation of the intermediate goods:

$$Y\left(t\right) = \left[\int y\left(z,t\right)^{\frac{\eta-1}{\eta}} dF\left(z,t\right)\right]^{\frac{\eta}{\eta-1}}$$

- A unit mass intermediate goods producers: choose "active" or "inactive"
  - Active firms produce and innovate:

$$y = \left(\left(1 - c\left(\mu\right)\right) e^{z}\right)^{\frac{1}{\eta-1}} \ell$$

  - Inactive firms try to find ideas
  - Let $\alpha\left(t\right)$ denote the mass of active firms in period t

Innovation and diffusion processes

- **Own Innovation**: innovator productivity follows

$$dz = \mu dt + \sigma dB$$

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- If imitator meets innovator z, draws a random quality $q \in \left[\underline{q}, \infty\right) \sim G\left(q\right)$:

$$z' = z + q$$

  - Learning may be limited if $q < 0$. Buera and Oberfield 2020
- **Creative Destruction**: with prob δ, imitator replaces innovator if $q > 0$
  - Incumbent replaced with probability $\hat{\delta} = \delta(1 - G(0))$
- **New Varieties**: with prob $1 - \delta$, imitator creates a new variety

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- A close mapping to many endogenous growth models

  Garcia-Macia Hsieh Klenow 2019 (GHK)

Firms' problem

- Innovator value $V(z, t)$ and imitator value $W(t)$
- Entrant surplus from imitation:

$$S^e(z, t) = \mathbb{E}_{q,\text{success}}\left[V(z+q, t) - W(t)\right]$$

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- Innovator and imitator values:

$$\begin{aligned} r(t)\,V(z,t) = \max_{\mu}\Big\{ & \Pi(t)\,(1-c(\mu))\,e^{z} + \mu V_z(z,t) + \frac{1}{2}\sigma^2 V_{zz}(z,t) + V_t(z,t) \\ & +\lambda(\alpha(t))\,\Big[\underbrace{\beta S^e(z,t)}_{\text{license income}} - \underbrace{\hat{\delta}\,(V(z,t) - W(t))}_{\text{creative destruction loss}}\Big]\Big\} \end{aligned}$$

$$r(t)\,W(t) = \iota(\alpha(t))\,(1-\beta)\,\mathbb{E}\left[S^e(z,t)\right] + W_t(t)$$

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- Threshold firm $\underline{z}(t)$:

$$V(\underline{z}(t),t) = W(t) \quad \text{and} \quad V_z(\underline{z}(t),t) = 0$$

Equilibrium

Equilibrium

An equilibrium consists of a profit aggregator function $\Pi(t)$, value functions $\{V(z,t), W(t)\}$, policy functions $\{\underline{z}(t), \mu(z,t)\}$, productivity distribution $F(z,t)$, and fraction of innovators $\alpha(t) = 1 - F(\underline{z}(t), t)$, such that

- The value functions and policy functions solve the HJB equations.
- Market clearing determines $\Pi(t)$
- The distribution $F(z,t)$ follows the corresponding KF equation
    - Imitation: entry
    - creative destruction: entry/exit
    - Stochastic growth moves mass around + exit (bad draws)
    - $\Pi'(t) < 0$ implies $\underline{z}'(t) > 0$ gives exit at boundary $\underline{z}(t)$

Balanced Growth Path

- Innovation and diffusion jointly imply:
  - Growth rate g
  - Pareto tail of stationary distribution ζ

Proposition: BGP

$$g = \frac{1}{1-\eta}\left[\underbrace{\mu + \frac{1}{2}\sigma^2}_{\text{own innovation}} + \underbrace{\lambda(\alpha)\,\hat{\delta}\left(\mathbb{E}_{q\geq 0}\left[e^q\right]-1\right)}_{\text{creative destruction}} + \underbrace{\lambda(\alpha)(1-\delta)\left(\mathbb{E}\left[e^q\right]-\frac{e^{\underline{z}}}{\mathbb{E}\left[e^z\right]}\right)}_{\text{new varities - exit}}\right]$$

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- Tail of distribution ζ affected by:
  - Higher volatility σ lowers ζ (thickens tail)
  - Diffusion is a source of mean reversion:
    $\Rightarrow$ more (and better) diffusion increases ζ (thins tail)

Calibration

- Innovation cost function: $c\left(\mu\right)=\frac{\gamma}{1+\frac{1}{\varepsilon}}\mu^{1+\frac{1}{\varepsilon}}$
- Matching function: $M\left(\alpha,1-\alpha\right)=\kappa\alpha^{\omega}\left(1-\alpha\right)^{1-\omega}$
- Quality distribution: $G\left(q\right)=1-e^{-\phi\left(q-\underline{q}\right)}$, $\forall q\geq\underline{q}$

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- Kankanhalli Kwan 2022: royalty rate 11.5% of net sales in licensing $\Rightarrow$ around half-half split of profit

Calibration

GHK: growth decomposition, 1983-2013

$$g = \frac{1}{1-\eta}\left[\underbrace{\mu + \frac{1}{2}\sigma^2}_{\substack{\text{own innovation} \\ 1.15\%}} + \underbrace{\lambda(\alpha)\,\hat{\delta}\left(\frac{\phi}{\phi-1} - 1\right)}_{\text{creative destruction}} + \underbrace{\lambda(\alpha)(1-\delta)\left(\frac{\phi}{\phi-1}e^{\phi\underline{q}} - \frac{e^{\underline{z}}}{\mathbb{E}\left[e^{z}\right]}\right)}_{\text{new varieties} - \text{ exit}}\right]$$

- Own innovation: variance $\sigma = 0.11$.

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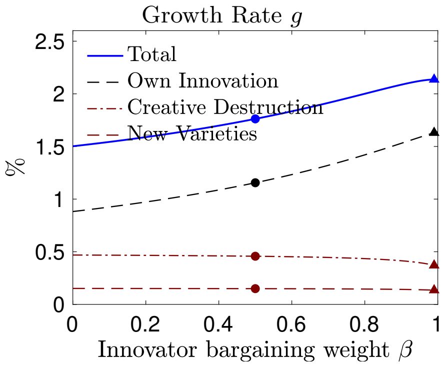
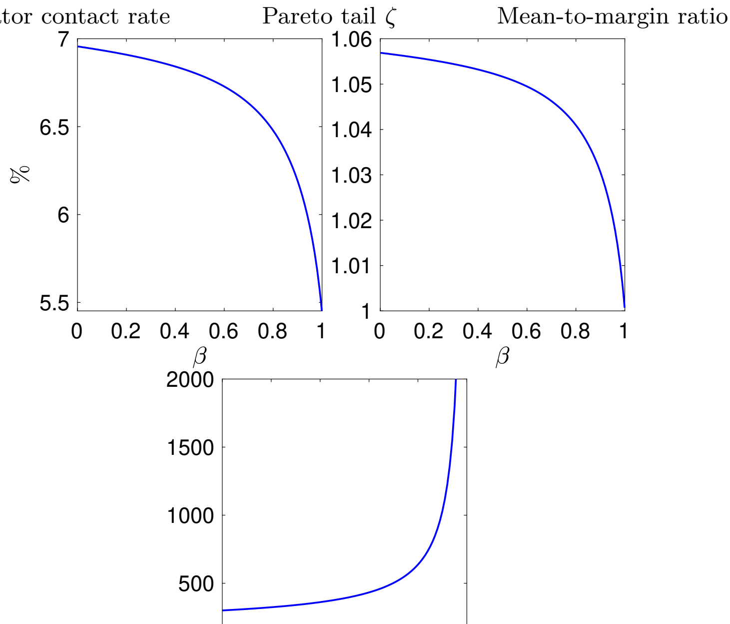
- Exit: marginal tiny compared to the mean

Appropriation and growth decomposition



- Strong effect of β on own innovation. Offsetting negative effect of creative destruction.
- Very weak reduction in diffusion

Appropriation and equilibrium heterogeneity



Optimal Policies

Ramsey Problem

$$W(F(z,0)) = \max_{\{\beta(t)\}_{t\geq 0}} \int_0^\infty e^{-\rho t} \log(C(t))\, dt$$

subject to:

$$Y(t) = \left[\int y(z,t)^{\frac{\eta-1}{\eta}} dF(z,t)\right]^{\frac{\eta}{\eta-1}}$$

$F(z,t)$ generated by equilibrium allocation starting from $F(z,0)$

- We characterize the optimal $\beta = \lim_{t\to\infty} \beta(t)$
- Limiting balanced growth path
- Different from long run growth maximizing

Main Forces

Optimal β satisfies:

$$
\begin{aligned}
0 = & \underbrace{\tilde{\pi}\left(\underline{z}, \mu\right) + d\tilde{V}\left(\underline{z}\right) + \left(M_1 - M_2\right)\mathbb{E}\left[\tilde{S}\left(z\right)\right]}_{\text{Congestion effect}} \\
& \underbrace{-\lambda\left(\alpha\right)\left\{\mathbb{E}\left[\tilde{V}\left(z\right)\right] - \tilde{V}\left(\underline{z}\right)\right\}}_{\text{Displacement externality}} \\
& + \underbrace{\lambda\left(\alpha\right)\frac{\psi^w}{1-\alpha}\left(-\left(1-\omega\right) - \left(\beta-\omega\right)\left(\frac{\alpha}{1-\alpha}\right)\right)\mathbb{E}\left[S\left(z\right)\right]}_{\text{Growth effect (non-appropriated)}}
\end{aligned}
$$

Same three forces as in the illustrative model

Preliminary Results - Special case

- Special case: perfet copying $z' = z$
- Exogenous innovation drift μ

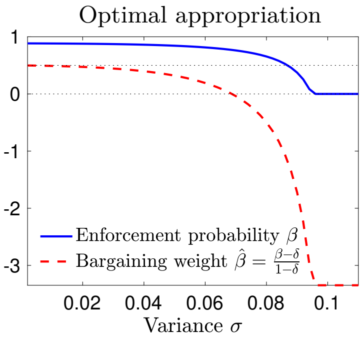
- Calibration: knowledge diffusion contributes to 0.62% growth

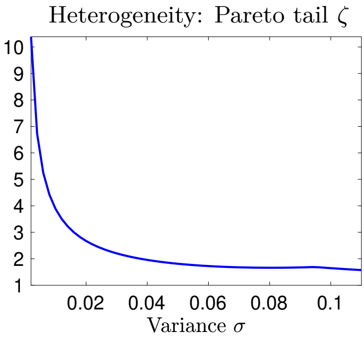
- Optimal enforcement probability β. Implied ex-ante bargaining weight

$$\frac{\beta - \delta}{1 - \delta}$$

- Displacement effect dominates: Enforcement probability zero!
- Still very preliminary

Heterogeneity: Randomness in innovation σ





Extension: alternative innovation process

- Other sources of shock

$$z = z_1 + z_2$$

  - Transferable component z_1 and non-transferable component z_2

$$dz_1 = \mu dt + \sigma_1 dB_1;\ dz_2 = \sigma_2 dB_2;\ \sigma^2 = \sigma_1^2 + \sigma_2^2$$

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- Experimentation and heterogeneity

$$dz = \mu dt + \sqrt{\mu}\sigma dB$$

  - Weaker incentives to innovate also reduces extent of heterogeneity

Conclusion

- Heterogeneity and growth through knowledge diffusion
- Baseline calibration suggests optimal share of surplus innovators fairly high
- Important impact of creative destruction
- Volatility matters a lot / not so much after recalibration

Appendix

Steady-state equilibrium ▸ Back

- $S^e(z) \equiv (1-\delta)\int_{\underline{z}-z}^{\infty}(V(z+q)-W)\,dG(q) + \delta\int_0^{\infty}(V(z+q)-W)\,dG(q)$

- Firm values satify HJB equations

$$\rho V(z) = \max_{\mu}\Big\{\pi(1-c(\mu))e^z + (\mu-(\eta-1)g)V_z(z) + \frac{1}{2}\sigma^2 V_{zz}(z) + \lambda(\alpha)\left[\beta S^e(z) - \tilde{\delta}(V(z)-W)\right]\Big\}$$

$$\rho W = \iota(\alpha)(1-\beta)\mathbb{E}\left[S^e(z)\right]$$

with boundary conditions

$$V(\underline{z}) = W \quad \text{and} \quad V_z(\underline{z}) = 0$$

- Distribution satisfies KF equation

$$\lambda(\alpha)\left\{(1-\delta)\int_{\underline{q}}^{z-\underline{z}} f(z-q)\,dG(q) + \delta\int_0^{z-\underline{z}}(f(z-q)-f(z))\,dG(q)\right\} + (\eta-1)gf'(z) - \frac{\partial(\mu(z)f(z))}{\partial z} + \frac{1}{2}\sigma^2 f''(z) = 0$$

Calibration

Parameter	Value	Moment	Data
Standard			
Discount rate ρ	0.032	Annual interest rate	5%
Elasticity of substitution η	4	Preset	
Innovation process			
Standard deviation σ	0.11	Own innovation randomness (GHK)	0.11
Cost function scale χ	629	Own innovation mean (GHK)	1.15%
Cost function elasticity ε	1	Preset	
Diffusion process			
Innovator bargaining weight β	0.5	Preset	
Matching function curvature ω	0.5	Preset	
Matching function efficiency κ	4.8	Total entry rate (GHK)	7.0%
Creative destruction probability $\tilde{\delta}$	0.77	Creative destruction rate (GHK)	5.4%
Quality draw shape ϕ	4.87	Quality draw shape (GHK)	4.87
Quality draw lower bound $\underline{q}$	-1.37	New variety average quality (GHK)	0.32