CAViaR models for Value at Risk and Expected Shortfall with long range dependency features

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Outline

Outline for section 1

Overview

- We consider alternative specifications of conditional autoregressive quantile models to estimate Value-at-Risk (VaR) and Expected Shortfall (ES)
- The proposed specifications include a slow moving component in the quantile process, along with aggregate returns from heterogeneous horizons as regressors
- Using data for ten stock indices, we evaluate the performance of the models and find that the proposed features are useful in capturing tail dynamics better

The advantages

- CAViaR models are useful for estimation of tail risk due to the stylised features of financial asset returns see *e.g.* among others [20, 3, 23, 19, 24, 21]
- CAViaR models have also been used to produce ES estimates based on joint elicitability arguments (among others [13])
- The need for costly recapitalisation could be decreased by reducing the portion of a realised loss when that loss exceeds the VaR (when the VaR series includes small gradual adjustments that come closer to anticipating the severity of future shocks)
- Past aggregate returns over multiple horizons could help obtain improved predictions of the severity of large losses
- We produce joint estimates of VaR and ES in a tractable manner (assuming a multiplicative relationship with an additional parameter in the style of [24])

Outline for section 2

VaR and ES definitions

Let $\{r_t\}_{t=1}^{T}$ denote the univariate time series of portfolio returns.

Definition 1 (VaR)

The VaR is given by

$$VaR_t = -F_t^{-1}(\theta)$$

where $\theta \in (0,1)$ is the specified quantile level, and $F_t(\cdot)$ represents the distribution of r_t conditional on time $t-1$ information.

Definition 2 (ES)

$$ES_t = -E_{t-1}[r_t | r_t \leq VaR_t]$$

A generic CAViaR model

$$q_{\theta,t}\left(\boldsymbol{\beta_\theta}\right) = \beta_{\theta,0} + \sum_{i=1}^{M} \beta_{\theta,i} q_{\theta,t-i}\left(\boldsymbol{\beta_\theta}\right) + \sum_{j=1}^{N} \beta_{\theta,M+j} l^j\left(\mathbf{x_{t-1}}\right) \tag{1}$$

- $q_{\theta,t}$, the θth quantile of r_t, is a function parameterised by $\boldsymbol{\beta_\theta}$, a vector with $M + N + 1$ parameters [1] [2]
- $\mathbf{x_t}$ contains the data observable at time t (including the history of returns $\{r_s\}_{s=0}^{t}$)
- $l^j(\cdot)$ are functions of lags of the data

[1] The usual practice is to set $M = 1$ as models of this order are found to work well for financial time series like the ones considered here.

[2] It is clear that the parameter vector $\boldsymbol{\beta_\theta}$ is specific to the quantile being modelled, and that the series of quantile estimates depend on the parameters.

Joint model for VaR and ES for a given quantile θ

Following [24],

$$q_t = \beta_0 + \beta_1 q_{t-1} + \beta_2 l(\mathbf{x}_{t-1})$$
$$e_t = (1 + \exp(\gamma))q_t \tag{2}$$

- e_t is the ES
- γ is a parameter that helps describe, in a simple but intuitive manner, the relationship between VaR and ES in the conditional return distribution

This approach to modelling ES is appealing for several reasons:

- Ensures that there are no crossings, *i.e.* the ES is always further out in the tail than the VaR
- The notion of a common scale parameter is generally supported by existing research among others [10, 26, 16, 24]
- The number of data points that are informative about the relationship between VaR and ES is quite small. So, it would be pragmatic to limit the number of parameters that depend crucially on this subset of observations for estimation

CAViaR specifications

- Symmetric Absolute Value (SAV):

$$q_t = \beta_0 + \beta_1 q_{t-1} + \beta_2 |r_{t-1}|, \tag{3}$$

- Asymmetric Slope (AS):

$$q_t = \beta_0 + \beta_1 q_{t-1} + \beta_2 r_{t-1}^+ + \beta_3 r_{t-1}^-, \tag{4}$$

where the notation employed is: $r^+ = \max(r, 0)$ and $r^- = -\min(r, 0)$

- Indirect GARCH (IG):

$$q_t = -\left(\beta_0 + \beta_1 q_{t-1}^2 + \beta_2 r_{t-1}^2\right)^{\frac{1}{2}}. \tag{5}$$

These models can each be extended to include ES following Equation 2. We can also extend these models by allowing for a time varying, slow moving component in the spirit of [7] and [11].

Component quantile models

- The component versions of CAViaR models that we propose below are written by reference to the SAV, AS and IG models, by replacing the intercept parameter β_0 with a time varying process that induces a long memory property to the VaR
- We propose models with two components that are motivated by the potential to capture long range dependence and to achieve a smoother series that could save transaction costs.
- We focus on the approach of [11], which has also been applied to option pricing by [5] and [4]

Component-AS model (C-ASd)

Consider for example the AS recursion in Equation 4, which can first be rewritten as:

$$q_t = u + \beta_1(q_{t-1} - u) + \beta_2 r_{t-1}^{+} + \beta_3 r_{t-1}^{-} \tag{6}$$

By allowing u to vary over time, we obtain the Component-AS model (C-ASd):

$$\begin{aligned} q_t &= u_t + \beta_1 \left(q_{t-1} - u_{t-1}\right) + \beta_2 r_{t-1}^{+} + \beta_3 r_{t-1}^{-} \\ u_t &= \beta_4 + \beta_5 u_{t-1} + \beta_6 r_{t-1} \end{aligned} \tag{7}$$

Component-SAV model (C-SAVd)

Similarly, we can define the other two component counterparts.
Component-SAV (C-SAVd):

$$
\begin{aligned}
q_t &= u_t + \beta_1(q_{t-1} - u_{t-1}) + \beta_2 \left| r_{t-1} \right| \\
u_t &= \beta_3 + \beta_4 u_{t-1} + \beta_5 r_{t-1}
\end{aligned} \tag{8}
$$

Component-IG model (C-IGd)

Component-IG (C-IGd):

$$
\begin{aligned}
q_t &= -\left\{ u_t^2 + \beta_1 \left(q_{t-1}^2 - u_{t-1}^2 \right) + \beta_2 r_{t-1}^2 \right\}^{\frac{1}{2}} \\
u_t &= \beta_3 + \beta_4 u_{t-1} + \beta_5 r_{t-1}
\end{aligned}
\tag{9}
$$

Component quantile models with multi-horizon returns: motivation

We also propose the incorporation of multi-horizon returns as regressors in these models to consider the possibility that past returns at lower than daily frequencies contain important information for risk estimation.
We want to answer:

- When we observe that the estimated VaR has not been exceeded for a long period of time, should we adjust our model?
- If so, should the adjustment lead to an increase or decrease in the VaR? This is a difficult question because it depends on one's view of the return process and on the loss function. This also has significant economic implications.

Time series models with multi-horizon returns: literature

The notion of incorporating information about different horizons has significant support in the statistics and finance literature.

- [15] document that monthly return volatility dynamics are different from daily dynamics
- [2] find that returns have dependence at different frequencies
- [25] find changes in the serial correlation in returns based on the volatility
- Among those who consider the impact of different frequencies or horizons in asset pricing are [1] and [9]. [9] find that future defaults are predicted by one year lagged stock return performance
- [8] use past returns among other variables to find credit default swap prices
- The model of [6] has been widely adopted as it has been shown to improve volatility forecasting by incorporating the history of realised volatility from different horizons. [18] use the HAR approach of [6] in the context of quantile forecasting using a location-scale model

Specifications

A simple approach to introduce long memory in our model is through the use of multi-horizon aggregate returns as regressors. Component-SAV multi-horizon (C-SAVdwm):

$$\begin{aligned} q_t &= u_t + \beta_1(q_{t-1} - u_{t-1}) + \beta_2 |r_{t-1}| \\ u_t &= \beta_3 + \beta_4 u_{t-1} + \beta_5 r_{t-1} + \beta_6 r^w_{t-1} + \beta_7 r^m_{t-1}, \end{aligned} \tag{10}$$

Component-AS multi-horizon (C-ASdwm):

$$\begin{aligned} q_t &= u_t + \beta_1(q_{t-1} - u_{t-1}) + \beta_2 r^+_{t-1} + \beta_3 r^-_{t-1} \\ u_t &= \beta_4 + \beta_5 u_{t-1} + \beta_6 r_{t-1} + \beta_7 r^w_{t-1} + \beta_8 r^m_{t-1} \end{aligned} \tag{11}$$

Component IG multi-horizon (C-IGdwm)

$$\begin{aligned} q_t &= -\left\{ u^2_t + \beta_1 \left(q^2_{t-1} - u^2_{t-1} \right) + \beta_2 r^2_{t-1} \right\}^{\frac{1}{2}} \\ u_t &= \beta_3 + \beta_4 u_{t-1} + \beta_5 r_{t-1} + \beta_6 r^w_{t-1} + \beta_7 r^m_{t-1}, \end{aligned} \tag{12}$$

where r^w and r^m represent aggregate returns over the past week (5 days) and month (22 days), respectively.

Interpreting the component structure

Unlike the traditional approach to writing component models, as discussed in detail in [5], here we have not attempted to write the model as a combination of two processes with zero-mean shocks.
Let us consider the AS model given in Equation (6). Assuming stationarity, if we take the unconditional expectation of the quantile, we get:

$$u = \overline{q} - \frac{\beta_2 \overline{r^+} + \beta_3 \overline{r^-}}{1 - \beta_1}, \tag{13}$$

where $\overline{q}$, $\overline{r^+}$, and $\overline{r^-}$ represent the expectations of the respective variables. Thus, we can see that, unlike the traditional approach to writing such models, here $u \neq \overline{q}$. This can also be seen by substituting u from Equation (13) into Equation (6) to retrieve a more intuitive version of the AS model.

$$q_t = \overline{q} + \beta_1(q_{t-1} - \overline{q}) + \beta_2 \left(r_{t-1}^+ - \overline{r^+}\right) + \beta_3 \left(r_{t-1}^- - \overline{r^-}\right)$$

Interpreting the component structure, cont.

Looking again at the Component-AS model in Equation 7, assuming stationarity and for simplicity that $\overline{r} = 0$, we can solve again for the unconditional expectation of the quantile $\bar{q}$ and the unconditional mean of the component $\bar{u}$:

$$\overline{q} = \frac{\beta_4}{1-\beta_5} + \frac{\beta_2 \overline{r^+} + \beta_3 \overline{r^-}}{1-\beta_1},$$

$$\text{where } \overline{u} = u = \frac{\beta_4}{1-\beta_5}$$

Thus, in the proposed component models, the deviation $q_t - u_t$ is the component that represents an adjusted distance from the unconditional mean $\overline{q}$, while the dynamics of u_t capture the time dependence in $\overline{q}$, albeit with an adjusted mean level. Our aim is to introduce longer range dependence in the quantile process via a more persistent component. As long as the persistence of the component u_t is higher than that of $q_t - u_t$ we are able to interpret it as a 'long-term' component; with the caveat that q_t tends to a quantity different from u_t in the long run.

Outline for section 3

Loss funtion

We estimate the models by minimising the FZ0 loss function derived by [21] based on the results of [13]:

$$L_{FZ0}(r, q, e; \theta) = -\frac{1}{\theta e} I_{r \leq q}(q - r) + \frac{q}{e} + log(-e) - 1 \qquad (14)$$

where $I(.)$ is an indicator function. This loss function is also appropriate for out of sample evaluation of the performance of the models.

Optimisation

- The optimisation surface for such problems is known to be multi-modal and generally problematic to minimise over.
- We use a relatively small set of starting values (based on predefined criteria) that take into account the relative scale of the parameters. More specifically, we select a series of "compatible" parameter inputs
- We first pick a grid of values for the autoregressive parameters in the model and pair each value with values from a grid of other (news impact) parameters
- In each case, we modify the β_0 parameter (or the corresponding constant term in the equation for component models) to be approximately consistent with the assumption that the quantile process is stationary around the in-sample unconditional quantile of the data
- Note that the parameter is not fixed - it is merely the starting value for the parameter that is chosen to be internally consistent for each combination of the other parameters

Example: SAV in Equation (3)

- We start by choosing a range of values for β_1, viz. $\{0.65, 0.8, 0.95\}$
- For each of these values, we only pair it with starting values of $\beta_2 \in -0.2, -0.1$
- We compute $\bar{q}$ as the relevant θth empirical quantile of the data (using only the in-sample data)
- Similarly, we compute the mean of the absolute value of the data, labelled $\bar{absr}$
- We can then imply starting values for β_0 using the formula $(1 - \beta_2)\bar{q} - \beta_3 \bar{absr}$, for each pair of values of β_2 and β_3 (assuming stationarity of the quantile time series)
- Thus, with only 6 sets of starting values, we obtain estimates that are as good or better than the original approach of evolutionary starting parameter selection in [12].

Outline for section 4

Data

- We compute daily log returns
- We remove days when the return is zero as we assume they are holidays
- The sample period ranges from January 01, 2010 to December 31, 2021
- From the start of the sample period we retain an in-sample estimation period of 1304 days (after cleaning the data of zero-return days)
- We reserve the period from May 2015 as the out of sample period, which represents a time of both up and down markets with varying levels of volatility and includes the Covid-19 pandemic
- We roll the sample forward one day at a time, re-estimating the one day ahead VaR and ES over the out of sample period
- $\theta = 0.01$

Additional benchmarks

- AR-IG ([20]) is included as it allows for serial correlation in returns and performed best among CAViaR models in a horse race in the same paper:

$$q_t = \alpha r_{t-1} - \left(\beta_0 + \beta_1 (q_{t-1} - \alpha r_{t-2})^2 + \beta_2 \left(\varepsilon_{t-1}\right)^2\right)^{\frac{1}{2}} \tag{15}$$

  where α is an AR(1) parameter in the return equation, such that μ_t in Equation (1) is replaced by αr_{t-1}. Note that in all other specifications, we retain the norm in the literature of setting $\mu_t = 0$

- IG-GJR (similar to ones proposed by [14] and [22]) and is used to capture asymmetry:

$$q_t = -\left(\beta_0 + \beta_1 q_{t-1}^2 + \beta_2 r_{t-1}^2 + \beta_3 r_{t-1}^2 I_{r_{t-1}<0}\right)^{\frac{1}{2}} \tag{16}$$

Finally we consider three popular parametric GARCH-type models (GARCH(1,1), EGARCH(1,1), and NGARCH(1,1))

Out of sample scores

	SAV	AS	IG	AR-IG	IG-GJR	C-SAVd	C-ASd	C-SAVdwm	C-ASdwm	C-IGd	C-IGdwm	GARCH	NGARCH	EGARCH
S&P500	1.058	0.975	1.035	0.979	1.013	0.967	0.967	1.060	0.975	**0.962**	0.973	1.219	1.333	1.240
SmallCap2000	1.252	1.218	1.242	1.221	1.228	1.212	1.216	1.254	1.217	**1.200**	1.220	1.934	1.944	2.103
DAX30	1.238	1.216	1.232	1.219	1.222	1.199	1.210	1.241	**1.185**	1.212	1.214	1.578	1.589	1.700
FTSE100	1.012	0.975	1.012	0.991	0.989	**0.959**	0.972	1.015	0.972	0.984	0.982	1.060	1.071	1.118
CAC40	1.295	1.265	1.282	1.270	1.270	1.254	1.255	1.297	1.259	**1.254**	1.260	1.504	1.556	1.557
EUROSTOXX50	1.333	1.296	1.317	1.297	1.301	1.283	1.285	1.334	1.289	**1.280**	1.289	1.505	1.553	1.563
SPTSX	0.792	0.736	0.798	0.766	0.762	**0.734**	0.736	0.797	0.737	0.757	0.757	0.991	0.969	0.951
NIKKEI225	1.442	1.406	1.457	1.430	1.437	**1.386**	1.404	1.443	1.404	1.430	1.432	1.651	1.719	1.711
HANGSENG	1.262	1.219	1.265	1.230	1.233	1.204	1.216	1.264	**1.183**	1.225	1.244	1.376	1.357	1.535
ATX	1.289	1.232	1.279	1.245	1.239	**1.220**	1.228	1.296	1.233	1.239	1.242	1.721	1.710	1.780
Rank	9.700	4.700	9.600	7.100	7.500	1.600	2.700	10.700	3.300	3.500	5.600	12.400	13.000	13.600

Note: This table presents the average value of the FZ0 loss function from daily one day ahead estimates over the out of sample period. Lower values are the best, so for each asset, the models are ranked from 1 to 14 in ascending order of the loss function value. In each row, we mark the best score(s) in bold. The best average rank (lowest number) is 1.6 for the C-SAVd model.

Unconditional Coverage (in %)

	SAV	AS	IG	AR-IG	IG-GJR	C-SAVd	C-ASd	C-SAVdwm	C-ASdwm	C-IGd	C-IGdwm	GARCH	NGARCH	EGARCH
S&P500	1.4	1.5	1.4	1.5	1.2	1.7	1.8	1.4	1.6	1.7	1.2	1.0	1.0	1.0
SmallCap2000	1.2	1.4	1.1	1.4	1.2	1.2	1.6	1.2	1.3	1.6	1.2	0.6	0.6	0.6
DAX30	1.6	1.6	1.7	1.7	1.6	1.5	2.1	1.6	1.6	1.3	1.5	0.5	0.6	0.4
FTSE100	1.4	1.5	1.4	1.5	1.5	1.6	1.3	1.5	1.5	1.3	1.4	0.7	0.9	0.6
CAC40	1.3	1.5	1.4	1.4	1.4	1.4	1.8	1.3	1.4	1.3	1.3	0.8	1.0	0.8
EUROSTOXX50	1.4	1.5	1.5	1.5	1.8	1.3	1.8	1.4	1.5	1.5	1.4	0.8	1.1	0.7
SPTSX	1.1	1.1	1.2	1.0	1.2	1.1	1.7	1.1	1.3	1.1	1.1	0.8	0.7	0.6
NIKKEI225	1.2	1.2	1.1	1.2	0.9	1.1	1.9	1.1	1.4	1.2	1.1	0.7	0.6	0.5
HANGSENG	1.2	1.3	1.5	1.6	1.3	1.4	1.7	1.2	1.8	1.5	1.5	0.6	0.7	0.5
ATX	1.0	1.4	1.1	1.3	1.1	1.6	2.0	1.1	1.4	1.3	1.2	0.5	0.7	0.5
NS count	10	10	9	9	9	9	4	10	9	10	10	10	10	9

Note: This table presents the unconditional coverage rate of the models in percentage points, for each of the models. A number closest to 1 represents the best unconditional coverage rate. The final row counts the number of times the unconditional coverage test was not significant at the 1% level for each model.

Skill scores

	SAV	AS	IG	AR-IG	IG-GJR	C-SAVd	C-ASd	C-SAVdwm	C-ASdwm	C-IGd	C-IGdwm	GARCH	NGARCH	EGARCH
S&P500	39.04	43.84	40.36	43.61	41.66	44.28	44.28	38.94	43.87	**44.59**	43.97	29.80	23.23	28.58
SmallCap2000	35.73	37.49	36.27	37.34	36.94	37.77	37.59	35.61	37.55	**38.40**	37.37	0.70	0.229	-7.95
DAX30	24.84	26.14	25.21	26.01	25.81	27.23	26.52	24.67	**28.06**	26.38	26.30	4.16	3.50	-3.23
FTSE100	34.65	37.05	34.65	35.99	36.14	**38.08**	37.22	34.46	37.23	36.45	36.59	31.54	30.85	27.76
CAC40	24.01	25.75	24.75	25.46	25.45	26.39	26.34	23.89	26.11	**26.44**	26.04	11.73	8.69	8.64
EUROSTOXX50	21.70	23.89	22.63	23.83	23.56	**24.63**	24.50	21.61	24.26	24.80	24.28	11.57	8.79	8.19
SPTSX	54.70	57.90	54.37	56.18	56.42	**57.98**	57.92	54.42	57.86	56.68	56.70	43.30	44.56	45.59
NIKKEI225	12.07	14.29	11.18	12.81	12.36	**15.51**	14.42	12.02	14.40	12.84	12.67	-0.66	-4.83	-4.33
HANGSENG	16.50	19.35	16.30	18.60	18.40	20.31	19.49	16.33	**21.71**	18.91	17.68	8.91	10.16	-1.59
ATX	30.03	33.12	30.59	32.44	32.75	**33.80**	33.34	29.68	33.06	32.78	32.62	6.62	7.20	3.40

Note: The skill score is reported relative to historical simulation. We take the score of the relevant model, divide it by the score of historical simulation using a window length of 1304 returns, then subtract this ratio from 1 and multiply the result by 100. The historical simulation benchmark is simply the 1% percentile of the past 1304 returns (window length). Thus, a bigger value is better. With bold we denote the best performing model.

Out of sample DQ test p-values for the 1% VaR

	SAV	AS	IG	AR-IG	IG-GJR	C-SAVd	C-ASd	C-SAVdwm	C-ASdwm	C-IGd	C-IGdwm	GARCH	NGARCH	EGARCH
S&P500	0.000	0.000	0.002	0.147	0.055	0.000	0.000	0.000	0.000	0.000	0.063	0.000	0.379	0.000
SmallCap2000	0.595	0.067	0.917	0.283	0.337	0.404	0.075	0.008	0.172	0.154	0.723	0.109	0.000	0.004
DAX30	0.000	0.002	0.001	0.001	0.002	0.008	0.000	0.000	0.063	0.010	0.000	0.028	0.056	0.003
FTSE100	0.000	0.152	0.003	0.207	0.160	0.004	0.251	0.000	0.187	0.004	0.005	0.187	0.943	0.062
CAC40	0.003	0.012	0.002	0.005	0.003	0.012	0.000	0.003	0.009	0.108	0.004	0.347	0.927	0.338
EUROSTOXX50	0.002	0.004	0.002	0.007	0.000	0.011	0.000	0.000	0.000	0.003	0.007	0.352	0.821	0.257
SPTSX	0.122	0.203	0.524	0.475	0.543	0.219	0.000	0.001	0.191	0.419	0.553	0.355	0.011	0.001
NIKKEI225	0.896	0.719	0.953	0.711	0.915	0.924	0.000	0.951	0.031	0.424	0.832	0.912	0.858	0.653
HANGSENG	0.321	0.420	0.000	0.000	0.054	0.401	0.000	0.187	0.004	0.004	0.013	0.032	0.190	0.012
ATX	0.539	0.439	0.631	0.515	0.619	0.127	0.000	0.087	0.358	0.557	0.610	0.030	0.003	0.011
%	50	70	60	60	70	70	20	30	60	60	60	90	80	60

Note: We report p-values for the DQ test, performed for the series of out of sample estimates. A low p-value (≤ 0.01) suggests model rejection.The final row reports the percentage of times a model was not rejected.

Summary of Diebold-Mariano tests

	SAV	AS	IG	AR-IG	IG-GJR	C-SAVd	C-ASd	C-SAVdwm	C-ASdwm	C-IGd	C-IGdwm	GARCH	NGARCH	EGARCH
SAV	-	10	7	10	10	10	10	0	10	10	10	0	0	0
AS	0	-	0	0	0	10	10	0	8	5	4	0	0	0
IG	2	10	-	10	10	10	10	3	10	10	10	0	0	0
AR-IG	0	10	0	-	3	10	10	0	10	10	9	0	0	0
IG-GJR	0	10	0	7	-	10	10	0	10	10	8	0	0	0
C-SAVd	0	0	0	0	0	-	1	0	2	4	0	0	0	0
C-ASd	0	0	0	0	0	9	-	1	3	4	0	0	0	0
C-SAVdwm	9	10	7	2	10	10	9	-	10	10	10	0	0	0
C-ASdwm	0	2	0	0	0	8	7	0	-	4	2	0	0	0
C-IGd	0	5	0	0	0	6	6	0	6	-	2	0	0	0
C-IGdwm	0	5	0	2	2	10	10	0	8	8	-	0	0	0
GARCH	10	10	10	10	10	10	10	10	10	10	10	-	1	9
NGARCH	10	10	10	10	10	10	10	10	10	10	10	9	-	9
EGARCH	10	10	10	10	10	10	10	10	10	10	10	1	1	-

Note: We computed pairwise *t*-statistics for the Diebold-Mariano test comparing the average losses over the out of sample period for the 14 different forecasting models. This table counts the sign of the t-statistic across the 10 assets. A positive value indicates that the row model has higher average loss than the column model, so the column model is superior. However, many of the test values are not statistically significant.

With the exception of the C-SAVdwm model, the other component models tend to outperform, and are challlenged only by the

AS model, suggesting that asymmetry is a key feature to include.

Mean Absolute Error around ES on exceedance dates

	SAV	AS	IG	AR-IG	IG-GJR	C-SAVd	C-ASd	C-SAVdwm	C-ASdwm	C-IGd	C-IGdwm	GARCH	NGARCH	EGARCH
S&P500	0.048	0.096	0.095	0.133	0.046	0.157	**0.043**	0.087	0.075	**0.043**	0.089	2.521	3.965	1.911
SmallCap2000	0.194	0.245	0.249	**0.147**	0.189	0.169	**0.147**	0.212	0.204	0.244	0.158	2.791	2.648	1.781
DAX30	**0.096**	0.158	0.148	0.113	0.1	0.11	0.098	0.145	0.127	0.118	0.123	3.662	3.845	2.956
FTSE100	0.089	0.099	0.132	0.089	0.091	**0.087**	0.108	0.112	0.113	0.103	0.136	1.136	1.316	1.094
CAC40	0.073	0.091	0.077	0.087	0.08	**0.046**	0.064	0.082	0.07	0.065	0.084	0.704	0.722	0.786
EUROSTOXX50	0.106	0.048	0.053	0.034	0.144	**.031**	0.051	0.05	0.032	0.065	0.093	0.873	0.973	0.703
SPTSX	0.152	0.159	0.229	0.117	0.137	0.146	**0.11**	0.152	0.139	0.153	0.176	1.35	1.747	1.148
NIKKEI225	0.086	0.064	0.148	**0.048**	0.083	0.186	0.052	0.054	0.07	0.117	0.079	1.23	1.427	1.149
HANGSENG	0.139	0.162	0.238	**0.12**	0.158	0.146	0.266	0.14	0.314	0.132	0.147	1.439	1.987	1.198
ATX	0.109	0.066	0.1	**0.051**	0.098	0.062	0.075	0.093	0.108	0.095	0.151	2.291	2.871	1.552

Note: We report the mean size of the difference between the realised return and ES on exceedance days (when $r_t < q_t$). The number is multiplied by 1000 for ease of reading. The smaller this number, the lower the need to raise extra capital immediately at the worst times.

Conclusion

- We propose and evaluate CAViaR models that have long range dependence features (through the introduction of a component structure) and capture the possibility of lower frequency data being informative in predicting extreme risks (through the inclusion of past aggregate returns at heterogeneous horizons as regressors in the component process)
- In the sense implied by [17], our approach is guided by making micro-prudential methods more consistent with macro-prudential goals
- The models are motivated by the need to allow VaR to change steadily while taking into account variation in the underlying mean level of VaR
- The intuition behind this motivation is that it may assist in controlling transaction costs and in reducing the severity of the extreme losses (relative to VaR) when VaR is exceeded

Conclusion, cont.

- We introduce a simple practically useful approach to selecting starting values for parameters in the estimation algorithm to ensure the starting values are approximately compatible with each other in the sense that they are consistent with stationarity of the quantile process
- By analysing a particularly challenging period that covered the Covid-19 pandemic , we have provided more evidence that CAViaR models in general are good at capturing the underlying quantile process. Although our models introduce more parameters, they perform very well out of sample, pointing to the usefulness of considering long range dependence among modelling choices for CAViaR models

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