Running Primary Deficits Forever in a Dynamically Efficient Economy: Feasibility and Optimality

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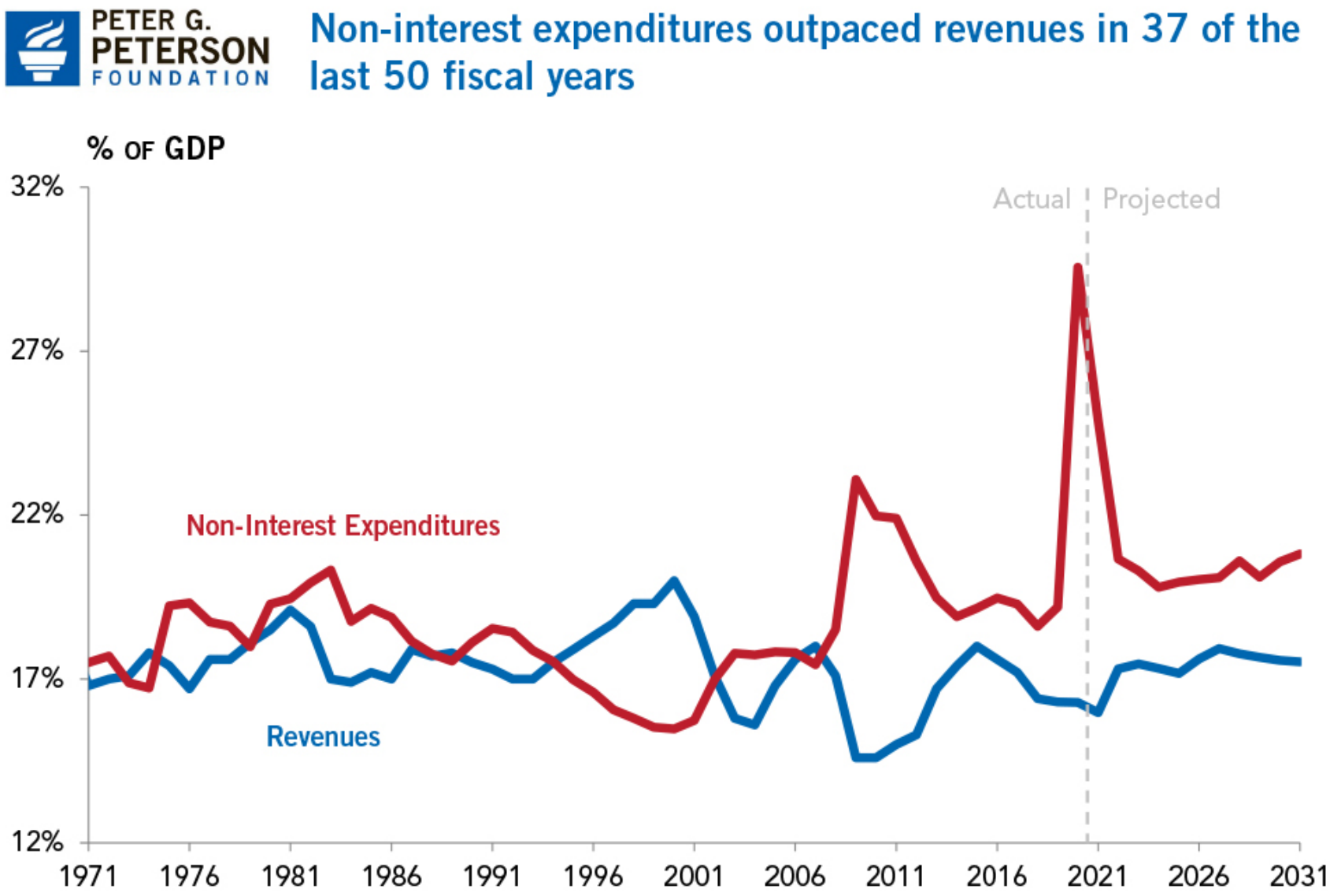
US Primary Deficit over the last 50 years



SOURCES: Congressional Budget Office, *The Budget and Economic Outlook: 2021 to 2031*, February 2021; and Office of Management and Budget, *Historical Tables, Budget of the United States Government: Fiscal Year 2021*, February 2020.

UK Primary Deficit over the last 50 years

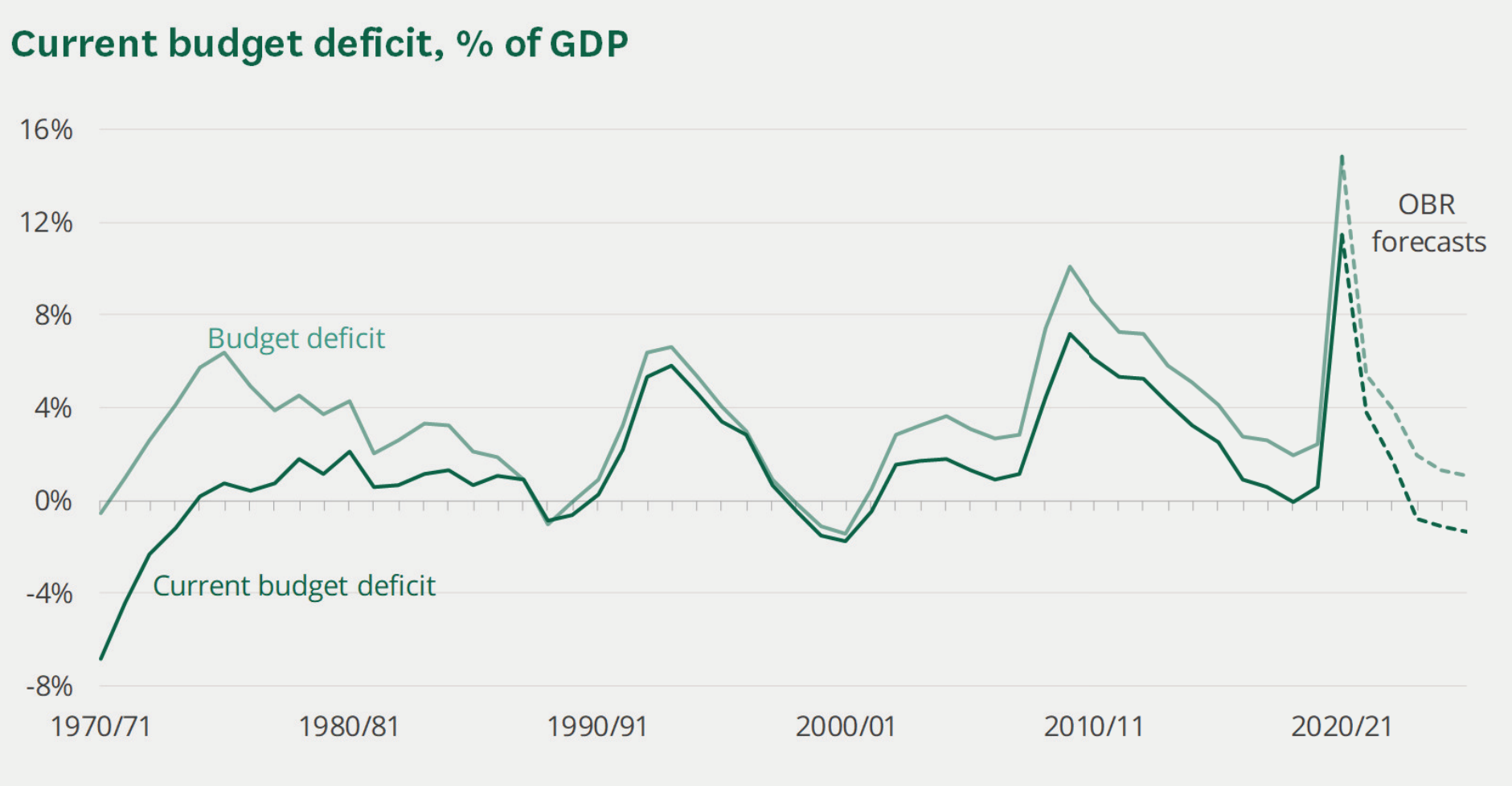


Figure: Source: House of Commons

G7 Primary deficits

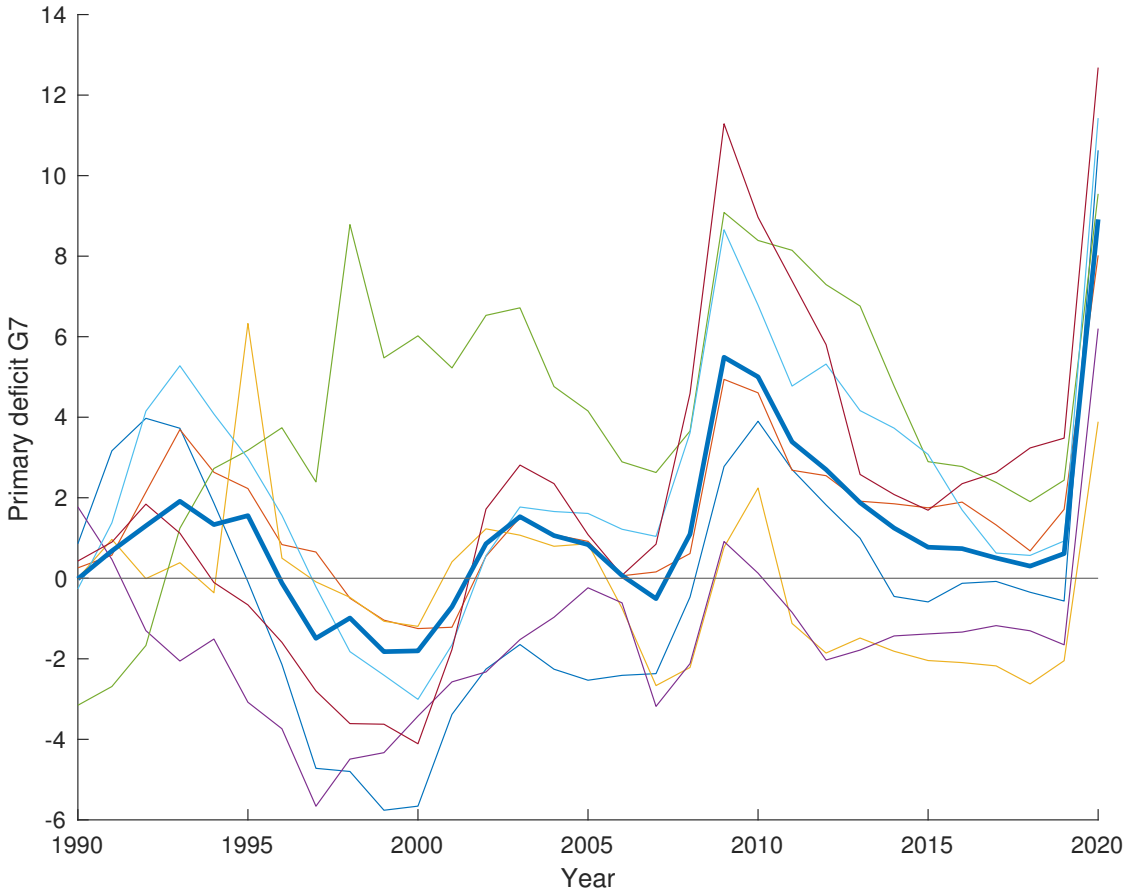


Figure: Source: "A Cross-Country Database of Fiscal Space", World Bank (2021). The thick blue line denotes the arithmetic average.

The decline in the real interest rate

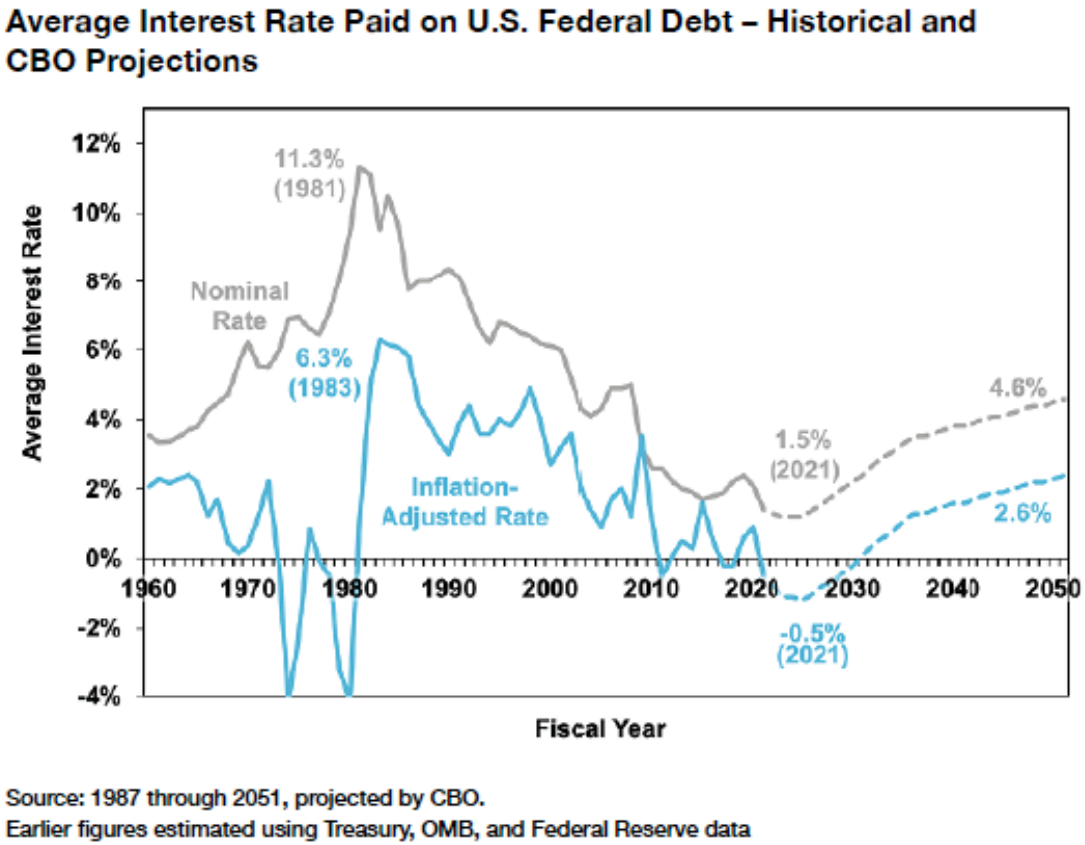


Figure: Source: Riedl B. (2021) "How Higher Interest Rates Could Push Washington Toward a Federal Debt Crisis"

Surplus decomposition

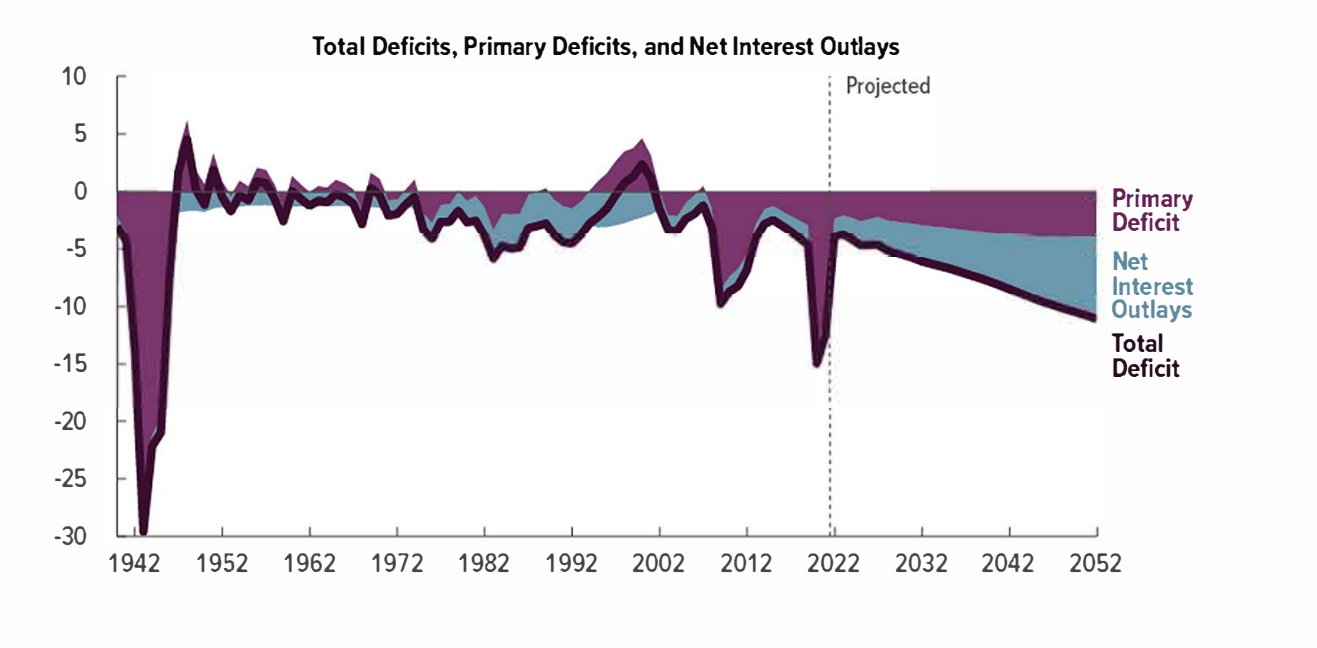


Figure: Source: CBO Website "The 2022 Long-Term Budget Outlook", July 2022

The outlook for long term debt

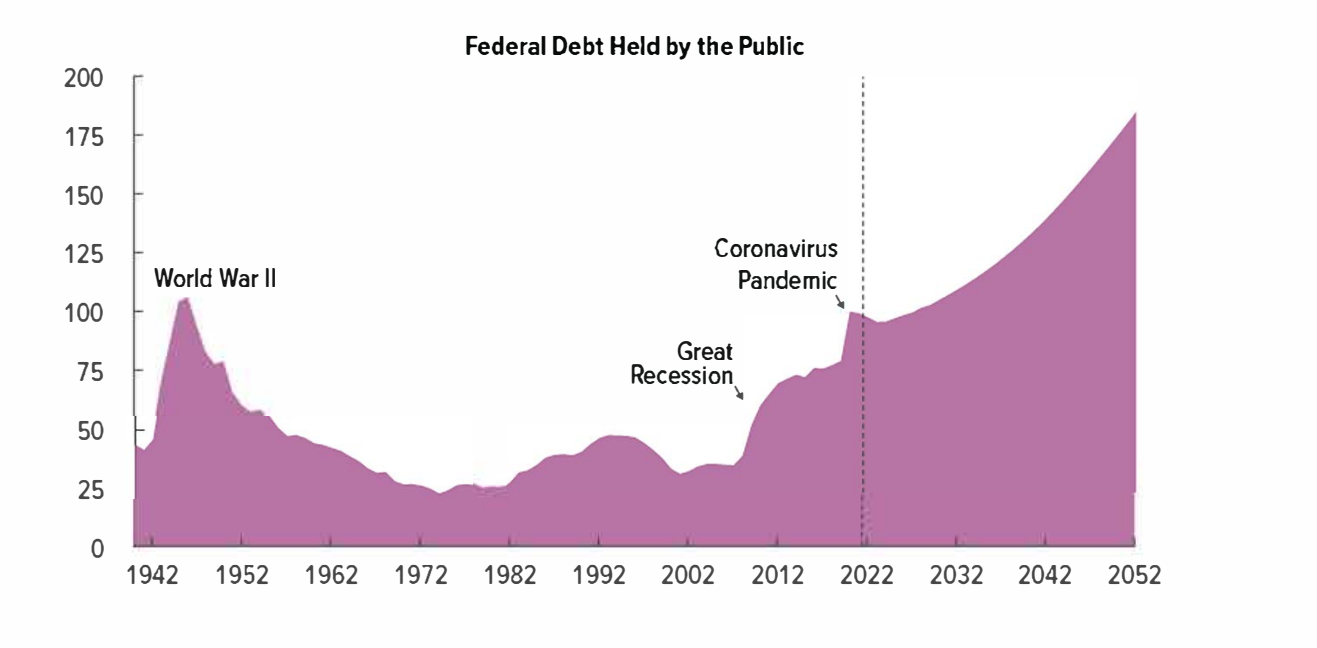


Figure: Source: CBO Website "The 2022 Long-Term Budget Outlook", July 2022

Role and Sustainabilty of Government Debt

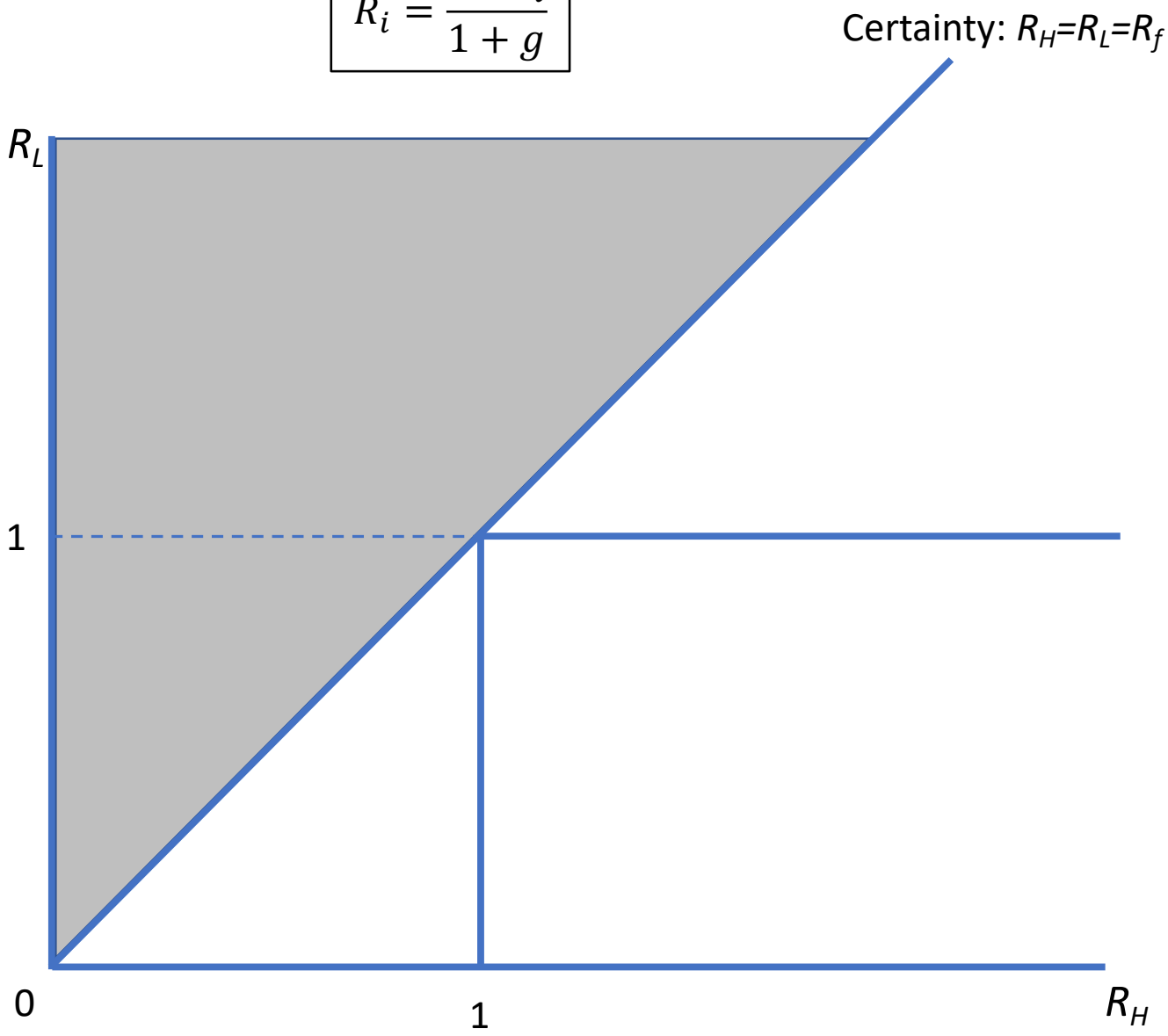
- Classic topic - Diamond (1965)
  - In an OLG model, debt can be rolled over without running primary surpluses
    - Deterministic setup
  - Common textbook intuition:
    - Over-accumulation of capital $r_f < g$ or $r_f + \delta < g + \delta$
    - Formalized (e.g., Cass (1974)) as the notion of "dynamic inefficiency"
    - In a stochastic setup, dynamic efficiency and debt-rollover feasibility are distinct.

The issue of dynamic efficiency

Cass's question of dynamic efficiency

- Fix some path of capital K_t and aggregate consumption C_t
- Is it possible to increase aggregate consumption in every period?
- Cass's thought experiment:
  - Capital decrement (consumption increment) by ε today, don't change consumption in future periods.
  - Next period capital is smaller by $\varepsilon(1 + r_{t+1})$.
  - ... After N periods $\varepsilon \prod_{i=1}^{N}(1 + r_{t+i})$.
  - The "undisturbed" capital would be $(1 + g)^N$. The ratio is $\varepsilon \prod_{i=1}^{N} \frac{(1+r_{t+i})}{1+g}$.
  - If this product converges to zero as $N \to \infty$, then in the long run, the capital decrement is like it never even happened.
- The Cass-Zilcha necessary and sufficient condition for dynamic efficiency. An economy is (in)efficient if $E \ln\{\frac{1+r}{1+g}\} = E\{\ln R\} \geq (<)0$.
- The issue of debt rollover is much simpler. It boils down to whether the risk-free rate, r_f, is below the growth rate of the economy, g.

$$R_i = \frac{1 + r_i}{1 + g}$$



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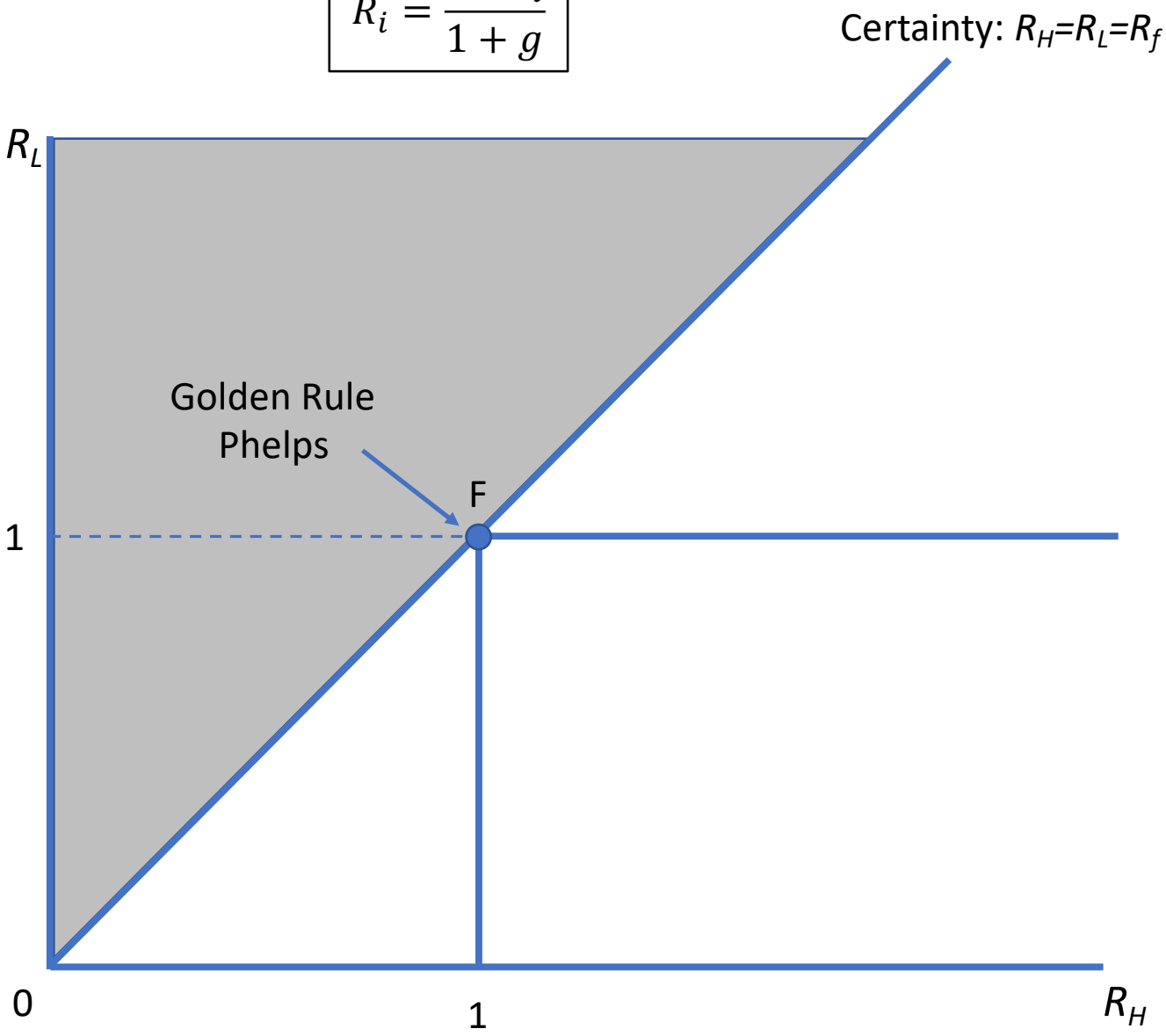

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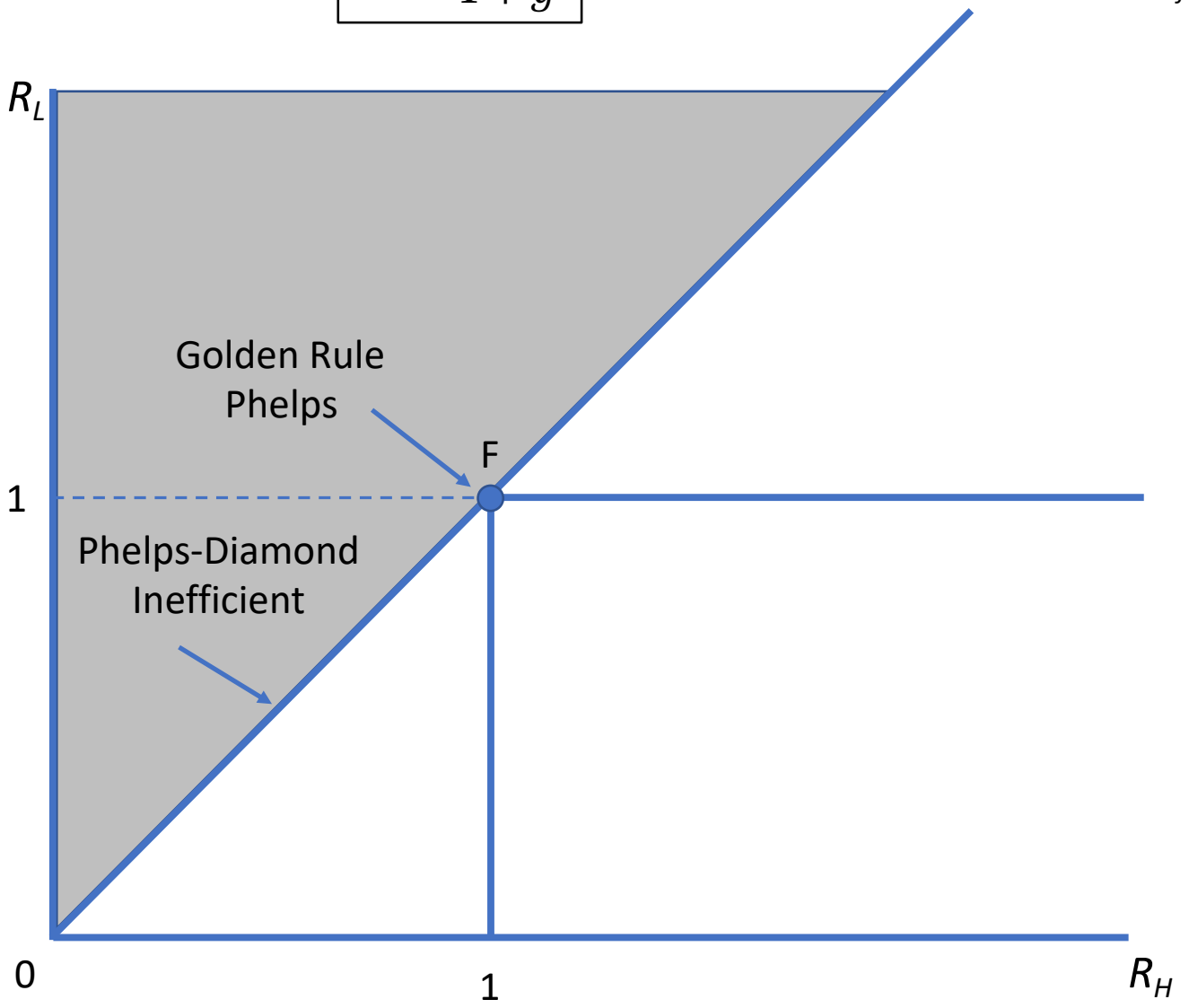

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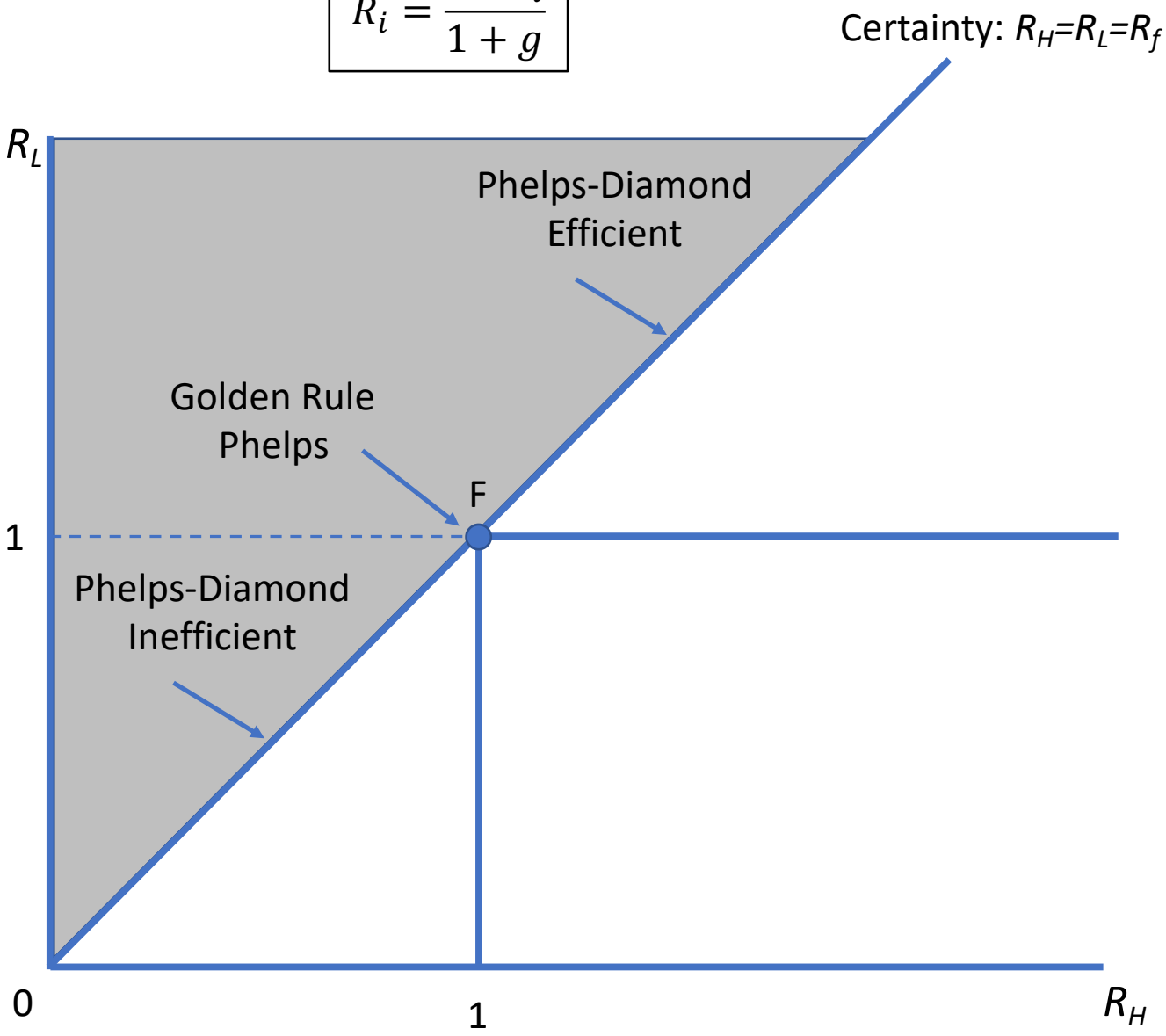

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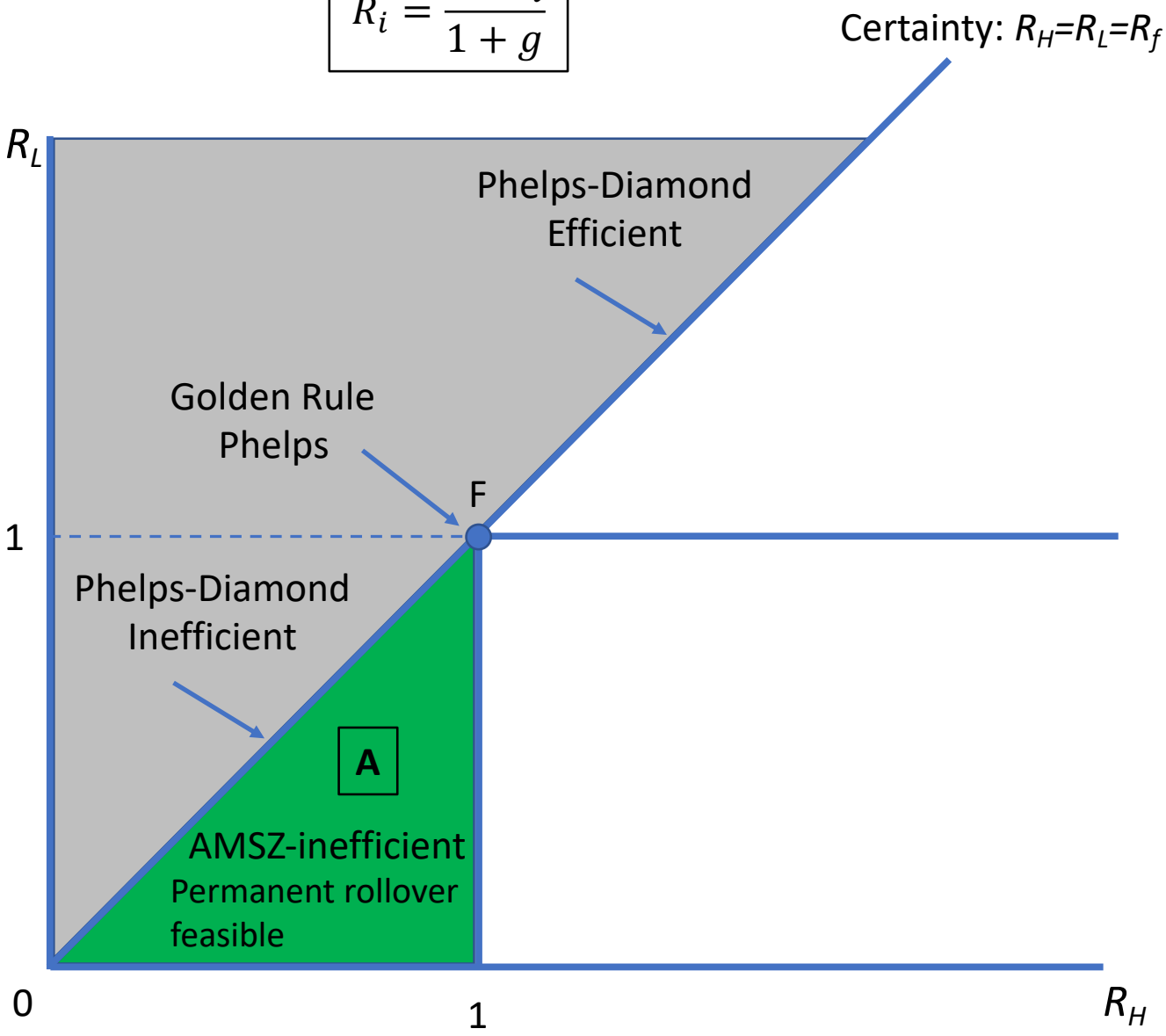

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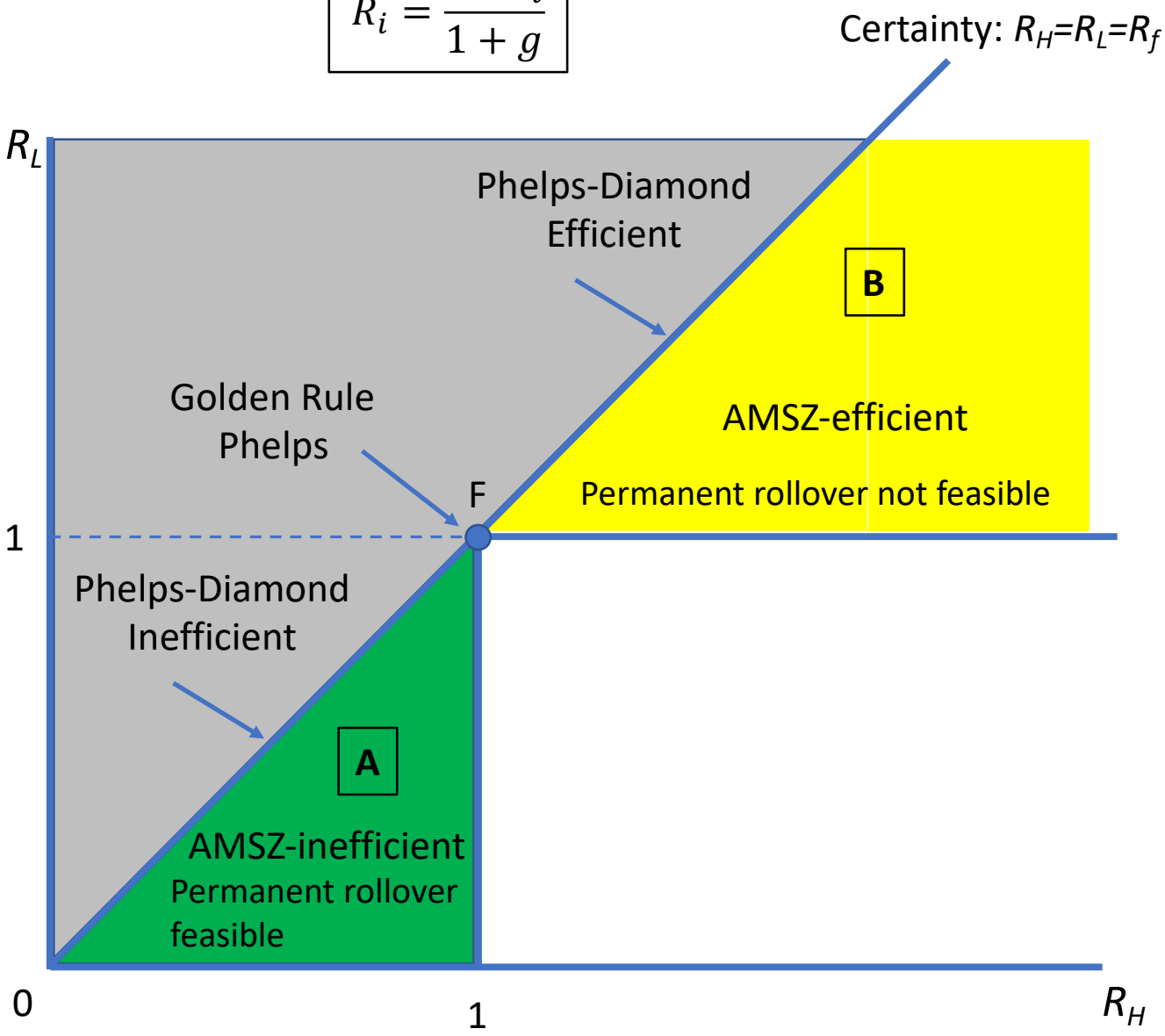

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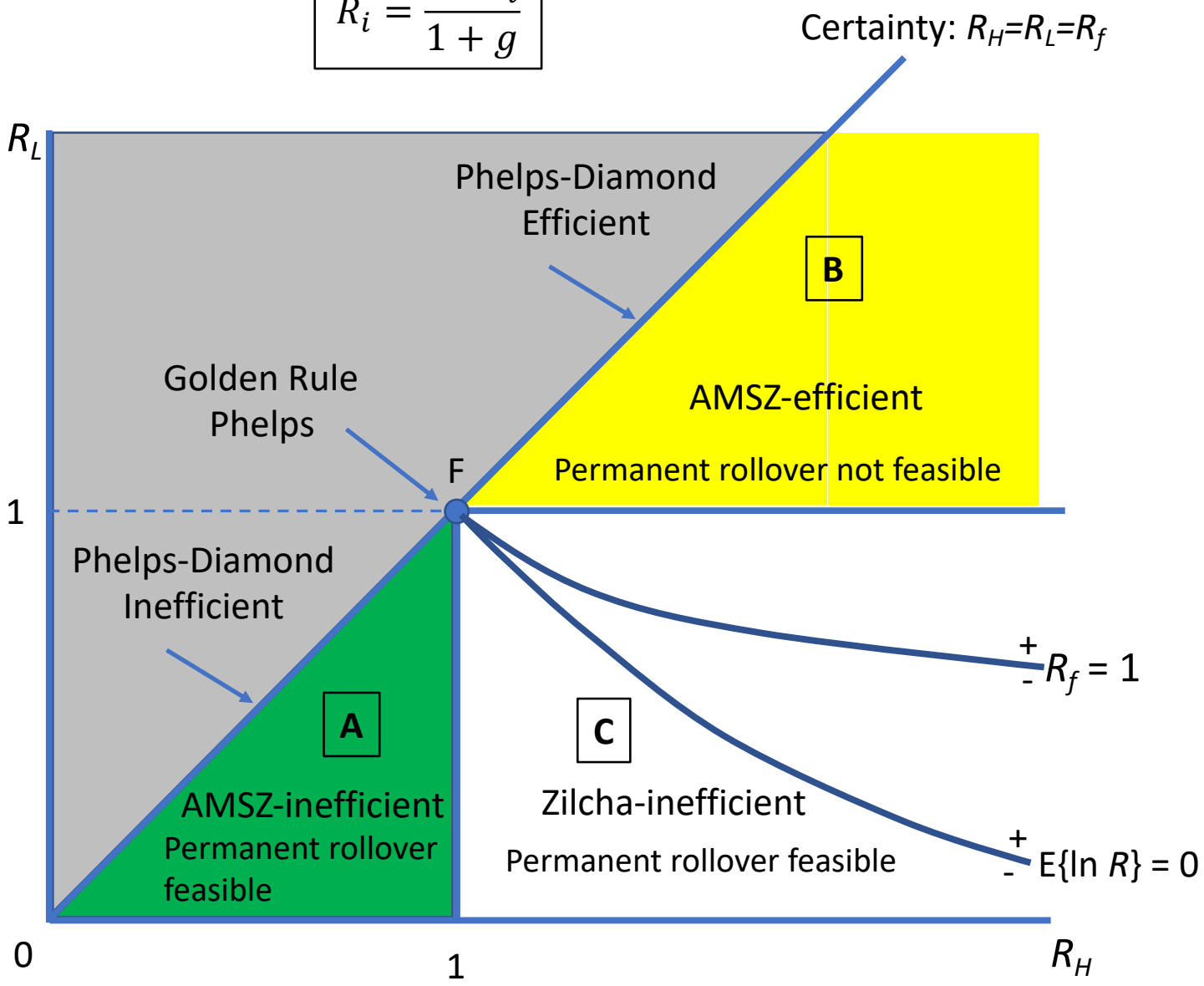

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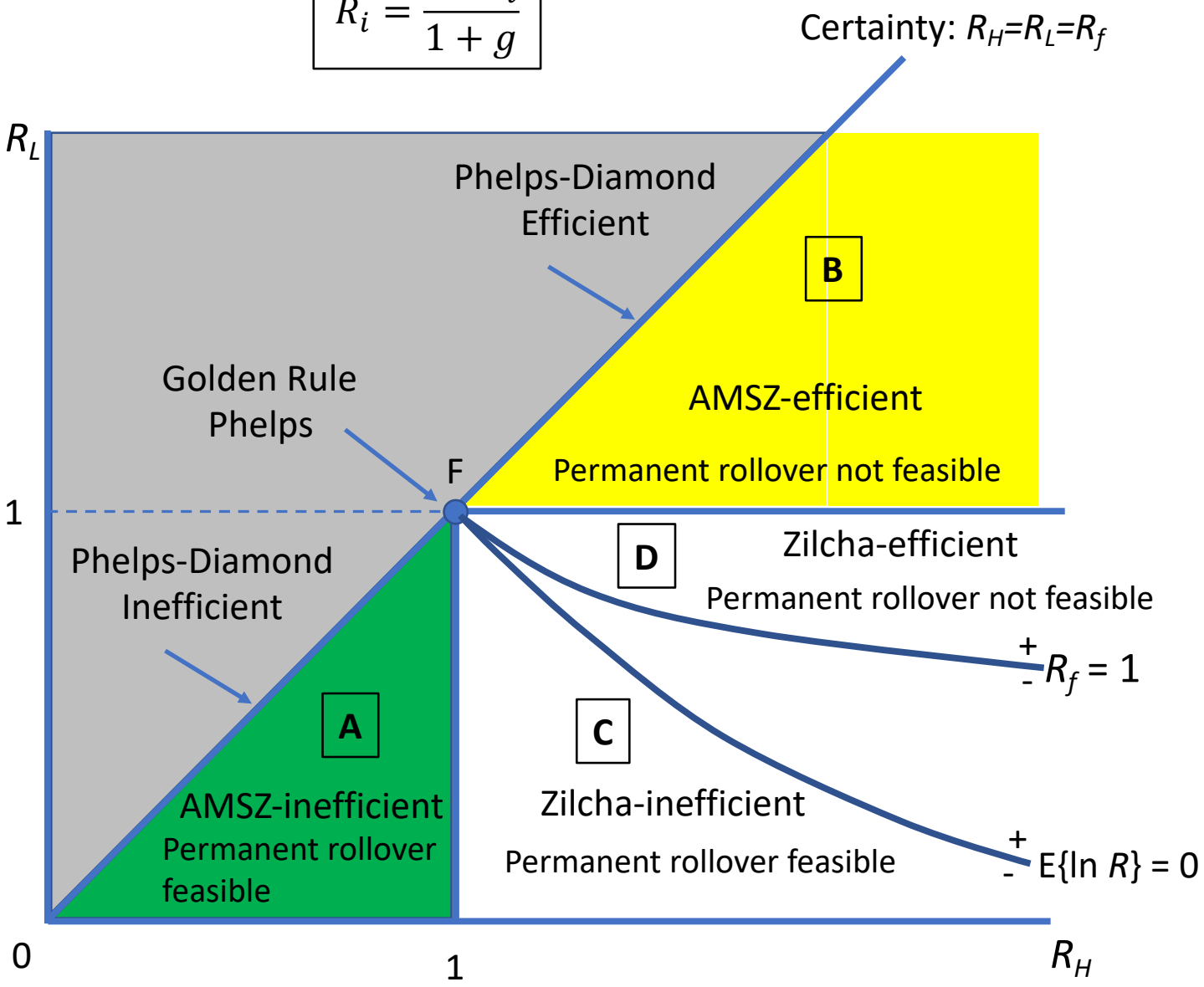

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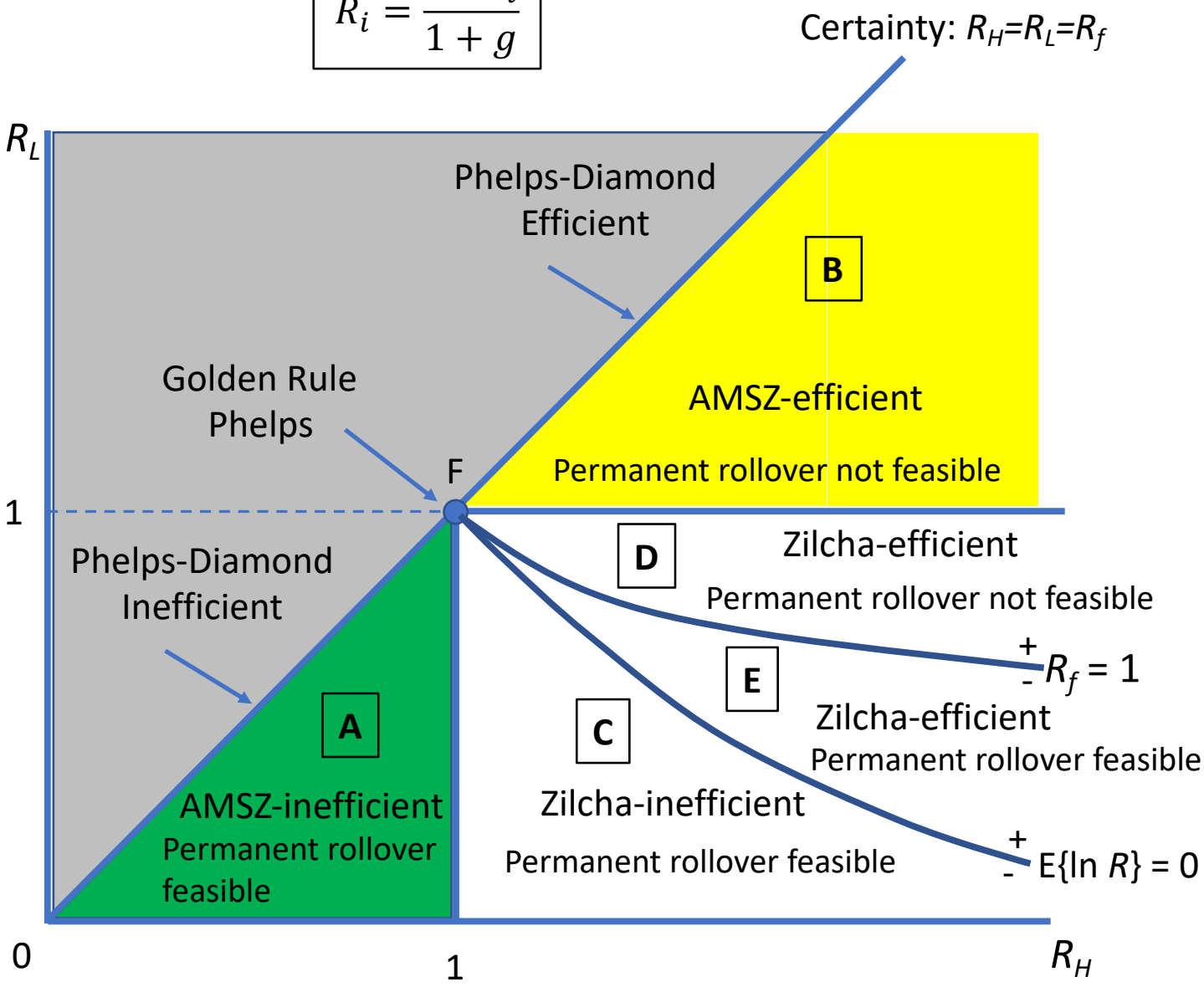

Results

1. Debt can be rolled over without surpluses, even if the economy is dynamically efficient.
   - The issue of "Over-accumulation of capital" (or dynamic efficiency) is distinct from the ability of the government to roll over debt without surpluses.
   - The question of debt rollover depends exclusively on whether $r_f < g$, which – if returns to capital are stochastic – can obtain in a dynamically efficient economy.
2. **Main result**: Steady-state utility is maximized by the debt to capital ratio for which $r_f = g$, which is the maximal sustainable ratio.
   - Debt allows intergenerational risk sharing.
   - But also "crowds out capital" in a dynamically efficient economy.
   - Despite this tradeoff, optimal debt = maximum sustainable debt.
   - Steady-state welfare is maximized when sufficient debt is issued so that $r_f = g$.

Framework

- Overlapping generations model with stochastic durability of capital
  - Designed to have convenient features
    - nonstochastic labor income, but stochastic return on capital (Bulow and Summers (1982))
    - Minimum deviation from Diamond (1965) to preserve tractability, but allows for a differentiation between the risk-free rate and a risky return on capital.
    - dichotomy between real and financial sectors in baseline case ($\zeta = 0$)
- Balanced growth path
  - Constant growth rates of capital and effective labor, g
  - Stochastic consumption growth of the marginal agent (risk premium)
  - Constant riskfree rate on government debt, r_f
    - can rollover forever iff $r_f \leq g$
  - i.i.d. stochastic rate of return on capital
    - easy to assess dynamic efficiency and distinguish between the possibility of debt rollover (without surpluses) and dynamic inefficiency

Production

- Cobb-Douglas production function

$$Y_t = F\left(K_t, G^t N\right) = \left(G^t N\right)^{1-\alpha} K_t^{\alpha}$$

  where $0 < \alpha < 1$ and $G \geq 1$

- Output per unit of effective labor

$$y_t = k_t^{\alpha}$$

  where $y_t \equiv \frac{Y_t}{G^t N}$ and $k_t \equiv \frac{K_t}{G^t N}$

- Depreciation rate of capital: $0 \leq \delta - \varepsilon_t \leq 1$ per period
  - Durability shock ε_t is i.i.d and has mean zero

Overlapping Generations

- N young consumers and N old consumers
- Each young consumer in period t
  - supplies G^t units of effective labor
  - earns wage income $W_t = (1-\alpha)\left(G^t\right)^{1-\alpha} K_t^{\alpha} N^{-\alpha} = (1-\alpha)\, G^t k_t^{\alpha}$
  - receives τ_t (receives transfer if $\tau_t > 0$; pays tax if $\tau_t < 0$)
  - saves $s_t = W_t + \tau_t - c_t^y$

Government Budget Accounting

- B_t one-period riskless government bonds at beginning of period t
- $D_t \geq 0$ is the primary government budget deficit during period t

$$B_{t+1} = (1 + r_{f,t})\, B_t + D_t$$

- Increase in bonds during period t is $g_{B,t} B_t$ so

$$B_{t+1} = (1 + g_{B,t})\, B_t$$

which implies

$$(g_{B,t} - r_{f,t})\, B_t = D_t$$

- Transfers to young generation: $N\tau_t = \zeta D_t$, where $0 \leq \zeta \leq 1$ and $(1 - \zeta)\, D_t$ is spent wastefully
- Transfer per young person

$$\tau_t = \zeta\, (g_{B,t} - r_{f,t})\, \frac{B_t}{N}$$

Old Consumers in Period t+1

- At beginning of period $t+1$, aggregate asset holdings

$$K_{t+1} + B_{t+1} = A_{t+1} = S_t = N s_t$$

- Aggregate capital stock in period $t+1$ is

$$K_{t+1} = A_{t+1} - B_{t+1} = S_t - B_{t+1}$$

- Net rate of return on capital purchased at the end of period t and used in period $t+1$ is

$$r_{t+1} = \alpha k_{t+1}^{\alpha-1} - \delta + \varepsilon_{t+1}$$

- Net rate of return on portfolio

$$r_{a,t+1} \equiv \lambda_{t+1} r_{f,t+1} + (1 - \lambda_{t+1})\, r_{t+1}$$

- Aggregate consumption of old

$$N c_{t+1}^{o} = (1 + r_{a,t+1})\, A_{t+1}$$

Utility Function

- Epstein-Zin-Weil utility function with $IES = 1$

$$\ln U_t = (1-\beta)\ln c_t^y + \beta \ln\left(\left[E_t\left\{\left(c_{t+1}^o\right)^{1-\gamma}\right\}\right]^{\frac{1}{1-\gamma}}\right)$$

- Use $s_t = W_t + \tau_t - c_t^y$ and $c_{t+1}^o = (1+r_{a,t+1})\, s_t$ to obtain

$$\begin{aligned}\ln U_t &= (1-\beta)\ln\left(W_t+\tau_t-s_t\right)+\beta\ln s_t \\ &\quad + \frac{\beta}{1-\gamma}\ln\left(E_t\left\{\left(1+r_{a,t+1}\right)^{1-\gamma}\right\}\right)\end{aligned}$$

- Optimal saving and consumption

$$s_t = \beta\left(W_t+\tau_t\right)$$

$$c_t^y = (1-\beta)\left(W_t+\tau_t\right)$$

$$c_{t+1}^o = \left(1+r_{a,t+1}\right)\beta\left(W_t+\tau_t\right)$$

Balanced Growth Paths (Steady States)

- Dynamic path of capital

$$k_{t+1} + \mathcal{B}_{t+1} k_{t+1} = G^{-1} \beta \left[(1-\alpha) k_t^{\alpha} + \zeta \left(g_{B,t} - r_{f,t} \right) \mathcal{B}_t k_t \right]$$

  where $\mathcal{B}_t \equiv \frac{B_t}{K_t}$

- Balanced growth path
  - K_t, Y_t, and B_t all grow at rate $g \geq 0$

$$G^{-1} \beta \left[(1-\alpha) k^{\alpha-1} + \zeta \left(g - r_f \right) \mathcal{B} \right] = 1 + \mathcal{B}$$

- Because IES=1: Dichotomy between the real and the financial sector. Risk aversion does not impact the determination of capital or output.

Steady-State Rates of Return

- Steady-state marginal product of capital is

$$\alpha k^{\alpha-1} = \frac{\alpha}{(1-\alpha)\,\beta}\left[(1+\mathcal{B})\,G - \beta\zeta\,(g - r_f)\,\mathcal{B}\right] \qquad \text{(constant)}$$

- Steady-state rate of return on capital is

$$r = \frac{\alpha}{(1-\alpha)\,\beta}\left[(1+\mathcal{B})\,G - \beta\zeta\,(g - r_f)\,\mathcal{B}\right] - \delta + \widetilde{\varepsilon} \qquad \text{(stochastic)}$$

Steady-State Adjusted Gross Rates of Return

- Use "adjusted gross rates of return"
  - $R \equiv \frac{1+r}{G}$ (capital)
  - $R_f \equiv \frac{1+r_f}{G}$ (riskfree government bonds)
  - $R_a \equiv \frac{1+r_{a,t+1}}{G} = \lambda R_f + (1-\lambda) R$ (portfolio of total assets)
- Expected and realized rate of return on capital

$$R = \overline{R}(\mathcal{B}, \zeta, R_f) + G^{-1}\widetilde{\varepsilon}$$

where

$$\overline{R}(\mathcal{B}, \zeta, R_f) \equiv \frac{\alpha}{(1-\alpha)\beta}\left[1 + \mathcal{B} - \beta\zeta(1-R_f)\mathcal{B}\right] + (1-\delta)G^{-1}$$

Isomorphic Production Environments

- Model in Paper:
  - Deterministic production function: $Y_t = (G^t N)^{1-\alpha} K_t^{\alpha}$
  - Stochastic depreciation: remaining capital $= (1 - \delta + \varepsilon_t) K_t$
  - **Deterministic evolution of output**
- Alternative Isomorphic Model
  - Stochastic production function: $Y_t = (G^t N)^{1-\alpha} K_t^{\alpha} + (1 - \delta + \varepsilon_t) K_t$
  - 100% depreciation per period
  - **Stochastic evolution of output**
- Class of Isomorphic Models
  - Stochastic production function: $Y_t = (G^t N)^{1-\alpha} K_t^{\alpha} + \eta_{1,t} K_t$
  - Stochastic depreciation: remaining capital $= (1 - \delta + \eta_{2,t}) K_t$
  - Equivalent to models above if $\varepsilon_t \equiv \eta_{1,t} + \eta_{2,t}$ has mean zero
  - **Stochastic evolution of output and depreciation**
- All models in this class have identical paths of:
  - capital, wage income, rates of return (risky and riskless)
  - $r_{f,t} - g_{B,t}$

Portfolio Allocation and Asset Pricing

- Share of riskfree government bonds in portfolio

$$\lambda_{t+1} = \arg\max_{\lambda_{t+1}} \frac{\beta}{1-\gamma} \ln E_t \left\{ \left(1 + r_{a,t+1}\right)^{1-\gamma} \right\}$$

- Recall $R_f \equiv \frac{1+r_f}{G}$ and $R \equiv \frac{1+r}{G}$

$$E\left\{ \left(\lambda R_f + (1-\lambda) R\right)^{-\gamma} \left(R_f - R\right) \right\} = 0$$

- Relation between optimal portfolio share and equilibrium bond-capital ratio

$$\lambda = \frac{B}{K+B} = \frac{\mathcal{B}}{1+\mathcal{B}}$$

Riskfree Interest Rate

Lemma

For any $\mathcal{B} \geq 0$ *and any distribution of* $R > 0$, $R_f(\mathcal{B}) = \dfrac{E_t\left\{R_a^{1-\gamma}\right\}}{E_t\left\{R_a^{-\gamma}\right\}}$.

- Note that risk aversion affects the determination of the risk-free rate and by implication the equity premium $r - r_f$.
- By adjusting risk aversion, we can affect the risk-free rate, without affecting capital accumulation.

Questions

1. When is this economy dynamically efficient?
   - Are the conditions required for dynamic efficiency empirically plausible?
2. Assuming the economy is dynamically efficient (so that increasing debt has an opportunity cost)
   - Is it possible to roll over debt without primary surpluses?
   - Is it optimal to do so and to what extent?

Sustainable Government Debt

Definition

A ratio of government bonds to the capital stock, $\mathcal{B}$, is *sustainable* if it can be rolled over forever at the riskfree interest rate without any primary budget surpluses in the future.

- A steady-state value of $\mathcal{B}$ will be sustainable if and only if the growth-adjusted, gross risk-free rate is below one, $R_f(\mathcal{B}) \leq 1$.

The issue of debt rollover

Dynamic efficiency depends on the sign of $E\{\ln R\}$.

- The possibility of debt rollover only depends on whether $R_f(\mathcal{B}) \leq 1$.
- Because of the dichotomy, the value of R_f depends on risk aversion, while the distribution of R does not depend on risk aversion.

Proposition

Assume that $\gamma \geq 1$, $\mathcal{B} = 0$, *and the adjusted gross rate return on capital,* $R \equiv \frac{1+r}{1+g} > 0$, *is a non-degenerate random variable.*

1. *If* $E\{\ln R\} \leq 0$, *then* $R_f < 1$.
2. *If* $R_f \geq 1$, *then* $E\{\ln R\} > 0$ *and the economy is dynamically efficient.*

... But it is well possible that $E\{\ln R\} \geq 0$ (dynamic efficiency) while $r_f \leq g$ (possibility of debt rollover).

Sustainable Government Debt

- A steady-state value of $\mathcal{B}$ will be sustainable if and only if $R_f(\mathcal{B}) \leq 1$.

Proposition

If $R_f'(\mathcal{B}) > 0$ *whenever* $R_f(\mathcal{B}) \leq 1$, *then*

1. *if* $R_f(0) \geq 1$, $\mathcal{B}_{\max} = 0$
2. *if* $R_f(0) < 1$, *then*
   1. $\mathcal{B}_{\max}$ *is the unique root of* $R_f(\mathcal{B}) = 1$
   2. $\mathcal{B}_{\max}$ *is finite*
   3. $\mathcal{B}$ *is sustainable if and only if* $0 \leq \mathcal{B} \leq \mathcal{B}_{\max}$

Balanced growth Utility

$$U_t = (1-\beta)\ln c_t^y + \beta \ln\left(\left[E_t\left\{\left(c_{t+1}^o\right)^{1-\gamma}\right\}\right]^{\frac{1}{1-\gamma}}\right)$$

- Optimal consumption when young and when old

$$c_t^y = (1-\beta)(W_t + \tau_t)$$

$$c_{t+1}^o = (1+r_{a,t+1})\beta(W_t+\tau_t)$$

- $U_t = \ln(W_t+\tau_t) + \frac{\beta}{1-\gamma}\ln E_t\left\{(1+r_{a,t+1})^{1-\gamma}\right\} + constant$
- Define $u = U_t - \ln G^t$ and $\ln w = \ln W_t - \ln G^t$

$$u = \ln w + \ln\left(1+\frac{\tau_t}{W_t}\right) + \frac{\beta}{1-\gamma}\ln E_t\left\{R_a^{1-\gamma}\right\} + constant$$

Optimal Sustainable Value of Bond-Capital Ratio

Proposition

If $R_f'(\mathcal{B}) > 0$ *whenever* $R_f(\mathcal{B}) \leq 1$, *then* $\arg\max_{[0,\mathcal{B}_{\max}]} u(\mathcal{B})$ *equals* $\mathcal{B}_{\max}$, *that is, utility per effective unit of labor along a balanced growth path is maximized by the maximum sustainable value of* $\mathcal{B}$.

Corollary

$\mathcal{B}_{\max}$ *is invariant to* ζ.

Heuristic Proof

Assume $G = 1$, $\zeta = 0$, so in long run, K, W, B, R_f, c_t^y are all constant

- General time-separable utility $U_t = u^y\left(c_t^y\right) + E_t\left\{u^o\left(c_{t+1}^o\right)\right\}$
- Use fundamental relationships in 3 areas of economics
  1. Envelope theorem from consumer theory
     - $\frac{dU_t}{d\mathcal{B}} = u^{y\prime}\left(c_t^y\right)\frac{dW_t}{d\mathcal{B}} + E_t\left\{u^{o\prime}\left(c_{t+1}^o\right)\right\}\left(\frac{K}{N}\frac{dR}{d\mathcal{B}} + \frac{B}{N}\frac{dR_f}{d\mathcal{B}}\right)$
  2. Factor-price frontier from neoclassical production theory, plus chain rule
     - $N\frac{dF_N}{dK} + K\frac{dF_K}{dK} = 0$, so $\frac{dW}{dK} = -\frac{K}{N}\frac{dR}{dK}$, and hence $\frac{dW}{d\mathcal{B}} = -\frac{K}{N}\frac{dR}{d\mathcal{B}}$
     - $\frac{du}{d\mathcal{B}} = u^{y\prime}\left(c_t^y\right)\left(-\frac{K}{N}\frac{dR}{d\mathcal{B}}\right) + E_t\left\{u^{o\prime}\left(c_{t+1}^o\right)\right\}\frac{K}{N}\frac{dR}{d\mathcal{B}} + E\left\{u^{o\prime}\left(c_{t+1}^o\right)\right\}\frac{B}{N}\frac{dR_f}{d\mathcal{B}}$
  3. $E\{MR\} = 1$ from asset pricing
     - $u^{y\prime}\left(c_t^y\right) = R_f E_t\left\{u^{o\prime}\left(c_{t+1}^o\right)\right\}$

$$\underset{(+)}{\frac{du}{d\mathcal{B}}} = \underset{(+)}{u^{y\prime}\left(c_t^y\right)}\left[\underbrace{\left(\frac{1}{R_f} - 1\right)}_{sign\left(1-R_f\right)}\underset{(+)}{\frac{K}{N}}\underset{(-)}{\frac{dR}{dK}}\underset{(-)}{\frac{dK}{d\mathcal{B}}} + \underbrace{\frac{1}{R_f}}_{(+)}\underset{(+)}{\frac{B}{N}}\underbrace{\frac{dR_f}{d\mathcal{B}}}_{(+,\text{ if } R_f \leq 1)}\right]$$

Quantitative Application

- $\alpha = 0.33$
- one period is 30 years
- $\beta = 0.353$, so $\left(\frac{\beta}{1-\beta}\right)^{1/30} = 0.98$ $(2\%$ annual discount rate)
- $G = 1.35$ $(1\%$ annual growth in labor-augmenting productivity)
- durability shock, ε, is a lognormal variable minus a constant
  - calibrated: when $\mathcal{B} = 0.5$, $E\{(1+r)\}$ matches a target value of $(1+m)^{30}$, and $sd\{(1+r)\}$ matches a target value of $s\sqrt{30}$
    - note that these are unlevered returns
- Annualized return on levered equity, r_L^A, when $D = 45\%$ of $D+E$
  - If $m = 3\%$ and $r_f^A = 0.8\%$, then $E\left\{r_L^A\right\} = 4.8\%$
  - If $s = 0.12$, $s.d.\left\{r_L^A\right\} = 21.8\%$

	$\zeta = 0$							
	Maximum sustainable $\mathcal{B}$				Dynamic efficiency boundary			
	$\gamma = 1$	$\gamma = 3$	$\gamma = 8$	$\gamma = 10$	$\gamma = 1$	$\gamma = 3$	$\gamma = 8$	$\gamma = 10$
$s = 0.02$								
$s = 0.04$			0.042	0.062				
$s = 0.06$		0.026	0.122	0.150				
$s = 0.08$		0.075	0.193	0.224				
$s = 0.10$	0.017	0.120	0.252	0.285				
$s = 0.12$	0.043	0.162	0.301	0.334				
$s = 0.14$	0.068	0.199	0.342	0.374	0.013	0.013	0.013	0.013
$s = 0.16$	0.091	0.232	0.376	0.408	0.031	0.031	0.031	0.031
$s = 0.18$	0.113	0.261	0.405	0.435	0.049	0.049	0.049	0.049
$s = 0.20$	0.133	0.286	0.429	0.459	0.066	0.066	0.066	0.066
$s = 0.22$	0.152	0.309	0.450	0.478	0.083	0.083	0.083	0.083

Table: Maximum sustainable $\mathcal{B}$ and the value of $\mathcal{B}$ delineating the boundary of dynamic efficiency. γ denotes risk aversion, and s is the annualized standard deviation of the return on capital in an economy with $\mathcal{B} = 0.5$. ζ is set to zero.

	$\zeta = 1$							
	Maximum sustainable $\mathcal{B}$				Dynamic efficiency boundary			
	$\gamma = 1$	$\gamma = 3$	$\gamma = 8$	$\gamma = 10$	$\gamma = 1$	$\gamma = 3$	$\gamma = 8$	$\gamma = 10$
$s = 0.02$								
$s = 0.04$			0.042	0.062				
$s = 0.06$		0.026	0.122	0.150				
$s = 0.08$		0.075	0.193	0.224				
$s = 0.10$	0.017	0.120	0.252	0.285				
$s = 0.12$	0.043	0.162	0.301	0.334				
$s = 0.14$	0.068	0.199	0.342	0.374	0.013	0.015	0.018	0.018
$s = 0.16$	0.091	0.232	0.376	0.408	0.032	0.037	0.043	0.044
$s = 0.18$	0.113	0.261	0.405	0.435	0.051	0.059	0.068	0.069
$s = 0.20$	0.133	0.286	0.429	0.459	0.069	0.080	0.091	0.092
$s = 0.22$	0.152	0.309	0.450	0.478	0.087	0.100	0.112	0.114

Table: Maximum sustainable $\mathcal{B}$ and the value of $\mathcal{B}$ delineating the boundary of dynamic efficiency. γ denotes risk aversion, and s is the annualized standard deviation of the return on capital in an economy with $\mathcal{B} = 0.5$. ζ is set to one.

Conclusions

- We have constructed a model in which governmnent bonds are riskless if $R_f \leq 1$, so the debt-capital ratio will not explode even if the government cannot run primary surpluses.
  - Can rollover debt forever if $R_f \leq 1$
    - Maximum sustainable debt, if positive, is attained when $R_f = 1$
    - Optimal sustainable debt equals the maximum sustainable debt
- What happens if equilibrium growth rate of capital is stochastic?
  - Sufficient condition to rollover debt forever: $R_{f,t} \leq 1$ in every period and every state
    - This condition is not necessary
    - Is there a maximum sustainable debt-capital ratio or some notion of maximal government borrowing?
    - If so, is that maximum sustainable amount optimal?