Long-Horizon Investing in a Non-CAPM World

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Motivation

Efficient Markets Hypothesis (EMH)

- EMH (Samuelson (1965), Fama (1970)):
  - Asset prices = Assets' expected future cashflows.
  - Asset returns are unpredictable.
- Profound impact of EMH:
  - Academic research.
    - Macro models embed implications of EMH, such as Expectations Hypothesis (EH) and Uncovered Interest Parity (UIP).
  - Financial regulation.
    - Mark-to-market regulation: Solvency of financial institutions is evaluated based on market value of their portfolios.
    - Laissez-faire approach to capital markets: Markets allocate capital efficiently towards firms/countries with best prospects.
  - Market practice.
    - Passive investing dominates active investing.
    - Active funds are evaluated based on market indices.

Violations of EMH

- While many empirical findings support EMH, there are robust violations.
  - Bonds.
    - Term-structure slope predicts positively excess returns of long- relative to short-term bonds, violating EH. (Fama-Bliss (1987), Campbell-Shiller (1991)).
  - Currencies.
    - Interest-rate differential between two countries predicts positively return of currency carry trade. Violation of UIP. (Bilson (1981), Fama (1984)).
  - Stocks:
    - Short-term momentum: Performance over the past 6-12 months continues on average over next year. (Jegadeesh-Titman (1993))
    - Long-term reversal: Performance over the past 3-5 years reverses on average over the next year. (DeBondt-Thaler (1985))
    - Value: stocks with high market value relative to book value earn low expected returns on average. (Fama-French (1992))
    - Value and momentum anomalies have been documented also for bonds, commodities and currencies. (Asness, Moskowitz and Pedersen (2013))

- How should implications of EMH for academic research, financial regulation and market practice be modified in inefficient markets?

This Paper

- Portfolio choice by long-horizon investors in inefficient markets.
  - Pension funds ($\approx$ \$60tn), insurance companies, sovereign-wealth funds, etc.

- Portfolio choice in efficient markets:
  - Hold riskless asset and market portfolio (CAPM).
  - Weight of market portfolio generally increases with investor horizon.
    Brennan-Schwartz-Lagnado (1997), Campbell-Viceira (1999,2002), Barberis (2000)

- Little is known about portfolio choice in inefficient markets, beyond general tools provided by portfolio theory.
  - Which market anomalies should investors hold?
  - How does optimal portfolio of anomalies depend on investor horizon?
  - What are main predictors of anomaly returns? Do they depend on horizon?

Equilibrium Approach

- Dynamic portfolio choice in inefficient markets involves many degrees of freedom.
  - How is each anomaly reflected in the cross-section of assets?
  - Which variables predict each anomaly's return?
  - How do anomaly returns correlate with each other?
  - How do anomaly returns correlate across time?

- Equilibrium approach $\rightarrow$ Tight and internally consistent specification.

- This paper:
  - Dynamic portfolio choice in inefficient markets using equilibrium approach.
  - Asset prices determined as in Vayanos-Woolley (RFS 2013, VW).
    - Capital moves slowly from underperforming to overperforming market segments.
    - **Value** and **momentum** anomalies.
  - Extend analysis in VW to derive properties of value and momentum returns that are key to dynamic portfolio choice.
  - Test properties empirically.
  - Derive implications for dynamic portfolio choice.

Results

- Moments of long-horizon returns differ significantly from short-horizon ones.
  - SR(value): Decreases with horizon for short horizons, then increases significantly.
  - SR(momentum): Decreases for short horizons, then is approximately flat.
  - Corr(value,momentum): Is negative for short horizons, then turns positive.

- $\rightarrow$ Optimal portfolio tilts from momentum to value as horizon increases.

- Conditional SR of value and momentum vary significantly over *flow cycle*.
  - SR(value): Negative at cycle's early stage (capital begins to leave underperforming segments), positive at late stage (capital has left).
  - SR(momentum): Positive at early and late stages, negative at intermediate stage.
  - Corr(value,mom): Predicts positively short-horizon SR of both strategies.
  - Value spread: Better predictor of long-horizon SR.

- $\rightarrow$ Misleading to use short-horizon metrics to guide long-horizon investing.

VW Summary

Model – Assets and Agents

- Continuous time $t \in [0, \infty)$.

- Assets.
  - Riskless rate r, exogenous.
  - N risky assets. Asset $n = 1, .., N$ pays dividend flow D_{nt}, trades at price S_{nt}, is in supply of η_n shares, and earns return per share $dR_{nt} = dD_{nt} + dS_{nt} - rS_{nt}dt$.

- Funds.
  - Index fund. Holds η_n shares of asset n. Value-weighted.
  - Active fund. Holds z_{nt} shares of asset n. Adds value because of hedgers.

- Agents.
  - Representative hedger.
    - Holds $\eta_n - \theta_n$ shares of asset n, not proportional to η_n.
    - θ_n is residual supply of asset n, to be held by remaining agents.
  - Representative investor.
    - Can invest in riskless asset, index fund, and active fund (y_t shares).
    - Exponential utility over intertemporal consumption, CARA $= \alpha$.
  - Representative active-fund manager.
    - Can invest in riskless asset and active fund ($\bar{y}_t = 1 - y_t$ shares). Smart money.
    - Exponential utility over intertemporal consumption, CARA $= \bar{\alpha}$.

Model – Frictions

- Investor suffers time-varying cost C_t from investing in active fund.
  - Reduced form for managerial ability.
  - Causes fund flows.

- Investor incurs flow cost $\frac{1}{2}\psi\left(\frac{dy_t}{dt}\right)^2$ to adjust her holdings in active fund.
  - Gradual adjustment.

- Dynamics of $D_t \equiv (D_{1t}, .., D_{Nt})$ and C_t:

$$\begin{aligned} dD_t &= F_t dt + \sigma dB_t^D, \\ dF_t &= \kappa(\bar{F} - F_t)dt + \phi\sigma dB_t^F, \\ dC_t &= \kappa(\bar{C} - C_t)dt + s dB_t^C, \end{aligned}$$

  where $(\kappa, \bar{F}, \bar{C}, \phi, \sigma)$ are constants and (B_t^D, B_t^F, B_t^C) are independent Brownian motions. Set $\Sigma \equiv \sigma\sigma'$.

- Investor observes only returns and share prices of index and active funds.
  - Conditional expectation of C_t is $\hat{C}_t$. Conditional variance is constant in steady state.

Equilibrium

- Dynamics of $\hat{C}_t$:

$$d\hat{C}_t = \kappa(\bar{C} - \hat{C}_t)dt - \beta_1 \left[p_f \left[dD_t - E_t(dD_t)\right] - (C_t - \hat{C}_t)dt\right] - \beta_2 p_f \left[dS_t - E_t(dS_t)\right],$$

where $p_f \equiv \theta - \frac{\eta\Sigma\theta'}{\eta\Sigma\eta'}\eta$ is *flow portfolio*.

  - $\hat{C}_t$ rises following underperformance of active relative to index fund.

- Investor's holdings of active fund:

$$v_t \equiv \frac{dy_t}{dt} = b_0 - b_1\hat{C}_t - b_2 y_t,$$

  - y_t drops gradually following increase in $\hat{C}_t$.

- Asset prices:

$$S_t = \frac{\bar{F}}{r} + \frac{F_t - \bar{F}}{r + \kappa} - \frac{\alpha\bar{\alpha}f}{\alpha + \bar{\alpha}}\frac{\eta\Sigma\theta'}{\eta\Sigma\eta'}\Sigma\eta' - (\gamma_0 + \gamma_1\hat{C}_t + \gamma_2 C_t + \gamma_3 y_t)\Sigma p_f',$$

where $(\gamma_0, \gamma_1, \gamma_2, \gamma_3)$ are constants.

  - Following increase in $\hat{C}_t$ or C_t, or decrease in y_t:
    - Assets that covary positively with p_f (overweighted by active fund) drop.
    - Assets that covary negatively with p_f (underweighted by active fund) rise.

Expected Returns

- Two-factor model:

$$\mathbb{E}_t(dR_t) = \frac{r\alpha\bar{\alpha}}{\alpha+\bar{\alpha}}\frac{\eta\Sigma\theta'}{\eta\Sigma\eta'}\mathbb{C}\mathrm{ov}_t(dR_t, \eta dR_t) + \Lambda_t \mathbb{C}\mathrm{ov}_t(dR_t, p_f dR_t).$$

  - Market portfolio η. Constant risk premium.
  - Flow portfolio p_f. Time-varying risk premium Λ_t, affine in $(\hat{C}_t, C_t, y_t)$.

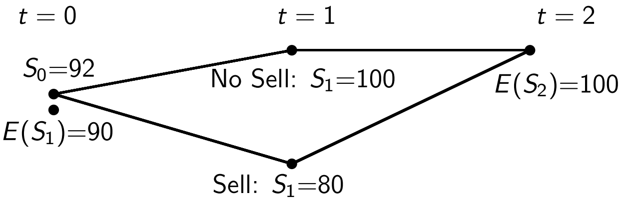
- **Flow cycle.**
  - Two market segments.
    - Active-fund overweights and underweights. Covariance with p_f.
  - Expected returns in each segment exhibit common variation. Following increase in $\hat{C}_t$, assets covarying positively (negatively) with p_f:
    - Immediate price drop (rise). Drop (rise) in expected return. **Anticipation** of flows. Return **momentum**.
    - Further price drop (rise). Rise (drop) in expected return. Flows **materialize**. Return **reversal**.

Bird-in-the-Hand Effect.

- Why does smart money absorb outflows? Wait to buy more cheaply after outflows materialize?

- Answer:
  - Price of assets covarying positively with p_f has dropped in anticipation of outflows.
  - $\rightarrow$ Buying those assets offers attractive long-run return.
  - Wait to buy more cheaply: risk that attractive return disappears.
  - Can restate in terms of Merton's intertemporal hedging demand ($\gamma > 1$).

- Example:



  - Buy at $t = 0 \rightarrow$ Lock-in expected return of 8.
  - Buy at $t = 1 \rightarrow$ Gamble for expected return of 20 or 0.

Trading Strategies and Calibration

Value and Momentum Weights

- Value weights:

$$w_t^V \equiv \left(\frac{\bar{F}}{r} + \frac{\epsilon(F_t - \bar{F})}{r+\kappa} - S_t \right)'.$$

  - Linear in difference between PV or expected future dividends and price.
  - Optimal ($\epsilon = 1$) or crude ($\epsilon = 0$) forecast.

- Momentum weights:

$$w_t^M \equiv \left(\int_{t-\tau}^{t} dR_u \right)'.$$

  - Linear in cumulative past return over interval $[t-\tau, t]$. Lookback window τ.

- Weight w_{nt} of asset $n \equiv$ Number of shares of asset n held by strategy.

Performance Measure

- Index adjust:

$$\hat{w}_t \equiv w_t - \frac{\mathbb{C}\mathrm{ov}_t(w_t dR_t, \eta dR_t)}{\mathbb{V}\mathrm{ar}_t(\eta dR_t)}\eta.$$

- Sharpe ratio (SR) of w_t over infinitesimal horizon dt:

$$SR_{w,t} \equiv \frac{\mathbb{E}_{\mathcal{I}_t}(\hat{w}_t dR_t)}{\sqrt{\mathbb{V}\mathrm{ar}_{\mathcal{I}_t}(\hat{w}_t dR_t) dt}}.$$

  SR over general finite horizon T:

$$SR_{w,t,T} \equiv \frac{\mathbb{E}_{\mathcal{I}_t}\left(\int_t^{t+T} \hat{w}_u dR_u\right)}{\sqrt{\mathbb{V}\mathrm{ar}_{\mathcal{I}_t}\left(\int_t^{t+T} \hat{w}_u dR_u\right) T}}.$$

  - Index adjust and annualize.
  - Condition on information set $\mathcal{I}_t$.
  - Unconditional SR ($\mathcal{I}_t = \emptyset$) denoted by SR_w and $SR_{w,T}$, respectively.
  - Continuous rebalancing.

- Compute all SR in closed form.

Performance over Infinitesimal Horizon

Optimal Strategy

- Invest in flow portfolio p_f proportionately to its risk premium Λ_t:

$$w_t = \Lambda_t p_f.$$

- Unconditional SR is 70.21%. For comparison, SR of index is 30%.

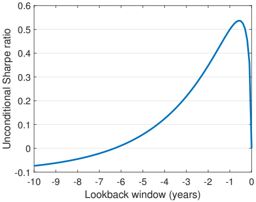
Value

- Unconditional SR is 26.88% for optimal-forecast strategy and 27.05% for crude-forecast one.
  - Focus on crude-forecast strategy.

- SR conditional on $\mathcal{I}_t = (\hat{C}_t, y_t)$:
  - Negative when $|\hat{C}_t|$ is high. Early stage of flow cycle.
    - High $\hat{C}_t$: Assets covarying positively with p_f experience price drop $\rightarrow$ Rise in value weight, and drop in expected return.
  - Positive when y_t adjusts to $\hat{C}_t$. Late stage.
    - Further price drop and rise in expected return.
  - Mean 21.46%, standard deviation 65.03%.

Momentum

- Unconditional SR is maximized at 53.66% for 7-month lookback window.
  - Focus on 7-month window from now on.



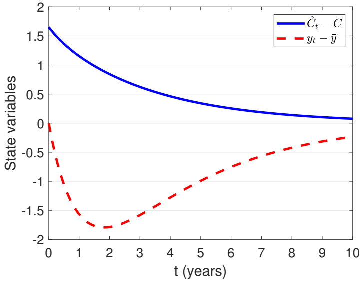
- SR conditional on $\mathcal{I}_t = (\hat{C}_t, y_t)$:
  - Positive when $|\hat{C}_t|$ is high. Early stage of flow cycle.
    - High $\hat{C}_t$: Assets covarying positively with p_f experience price drop $\rightarrow$ Drop in momentum weight, and drop in expected return.
  - Negative when adjustment of y_t is mid-way. Intermediate stage.
    - Further price drop and rise in expected return.
  - Positive when adjustment of y_t is full. Late stage.
    - Recent return history catches up with high expected return.
  - Mean 40.74%, standard deviation 46.58%.

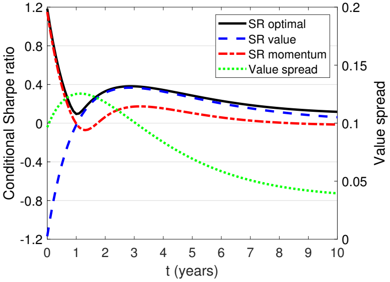
Value Spread

- Value spread $\equiv$ Standard deviation of market to book in cross-section of assets.
- Value spread conditional on $\mathcal{I}_t = (\hat{C}_t, y_t)$:
  - High when $|\hat{C}_t|$ is high. Early stage of flow cycle.
    - High $\hat{C}_t$: Assets covarying positively with p_f experience price drop and assets covarying negatively experience price rise.
  - Even higher when y_t adjusts to $\hat{C}_t$. Late stage.
    - Price wedge increases.
- Value spread as a return predictor.
  - Correlation between conditional value spread and SR of value is 26.00%.
  - Correlation between conditional value spread and SR of momentum is -8.13%.

Impulse Response

- Conditional value spread and SR of optimal strategy, value and momentum, following increase in $\hat{C}_t$:



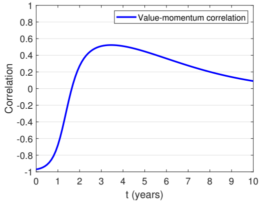


Combining Value and Momentum

- Unconditional value-momentum correlation is -12.20%.
  - Shocks to value and momentum returns are negatively correlated.
    - Value holds assets with long period of underperformance. Momentum tends to short these assets.
  - Conditional SR of value and momentum are positively correlated (28.22%).

- Unconditional SR of optimal value-momentum combination is 63.45%. For comparison, SR of optimal strategy is 70.21%.

- Conditional value-momentum correlation following increase in $\hat{C}_t$:

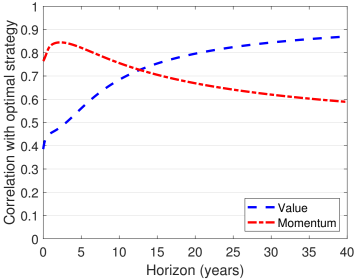


- Value-momentum correlation as a return predictor.
  - Correlation between conditional val-mom corr and SR of value is 86.02%.
  - Correlation between conditional val-mom corr and SR of mom is 41.34%.

Performance over General Finite Horizon

Optimal Strategy

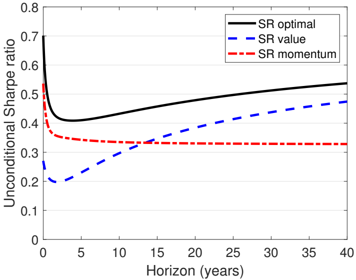
- Maximize $SR_{w,T}$ over strategies of the form $w_t = (\delta_0 + \delta_1 \hat{C}_t + \delta_2 C_t + \delta_3 y_t) p_f$.
- Correlation between optimal strategy and value/momentum:



- Optimal strategy is closer to momentum for short horizons and to value for longer horizons.

Unconditional SR

- Unconditional SR of optimal strategy, value and momentum:



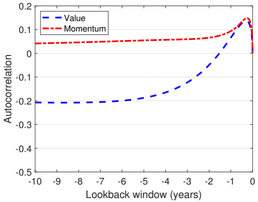
- Effect of horizon on SR is through autocorrelation of strategy returns.

$$SR_{w,T} = \frac{SR_w}{\sqrt{1 + 2\int_t^{t+T} \left(1 - \frac{u-t}{T}\right) \frac{\mathbb{C}\mathrm{ov}(\hat{w}_t dR_t, \hat{w}_u dR_u)}{\mathbb{V}\mathrm{ar}(\hat{w}_t dR_t)}}}.$$

  - Negative autocorrelation: SR increases with horizon.
  - Positive autocorrelation: SR decreases with horizon.

Autocorrelation

- Autocorrelation of value and momentum returns, one-year horizon:



- Value:
  - Positive autocorrelation over short horizons.
    - Asset-level momentum.
  - Negative autocorrelation over long horizons.
    - Low turnover of value combined with asset-level reversal.
    - Assets held by value experience further long period of underperformance $\rightarrow$ Expected returns *and* value weights rise.

- Momentum:
  - Positive autocorrelation over short horizons.
    - Asset-level momentum.
  - Zero autocorrelation over long horizons.
    - High turnover of momentum.

Conditional SR and Return Predictability (cont'd)

- Correlation between conditional value spread and SR:

	dt	1Yr	5Yrs	10Yrs	20Yrs
Value	26.00%	77.24%	97.81%	96.92%	96.45%
Momentum	-8.13%	27.13%	87.99%	91.75%	91.53%

Value spread becomes a better predictor as horizon increases.

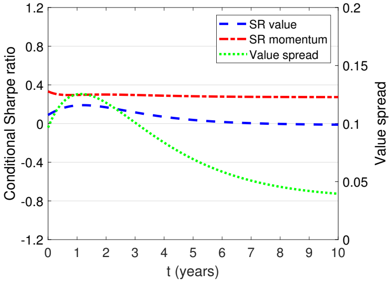
- Correlation between conditional value-momentum correlation and SR:

	dt	1Yr	5Yrs	10Yrs	20Yrs
Value	86.02%	47.63%	-3.56%	-7.72%	-6.92%
Momentum	41.34%	53.91%	18.47%	8.01%	6.64%

Value-momentum correlation becomes a worse predictor as horizon increases.

Impulse Response

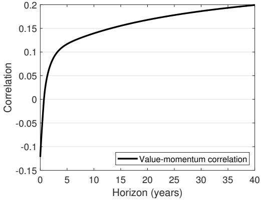
- Conditional value spread and five-year SR of value and momentum, following increase in $\hat{C}_t$:



- SR of value *rises*. SR of momentum barely moves.

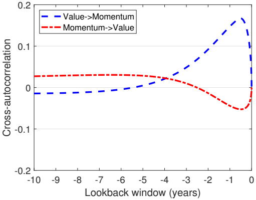
Combining Value and Momentum

- Value-momentum correlation:



- Switches from negative to positive as horizon increases.

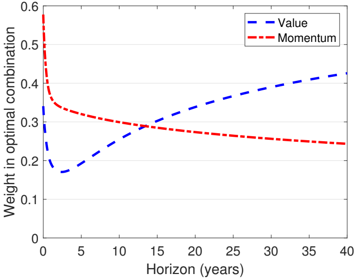
- Cross-autocorrelation of value and momentum returns, one-year horizon:



- Positive lead-lag effect from value to momentum.
  - Value earns high expected return towards end of flow cycle.
  - As high returns cumulate, momentum earns high expected return as well.

Optimal Portfolio

- Weights in optimal (SR maximizing) value-momentum combination:



- Weight of value relative to momentum rises threefold SR as horizon increases.

- Optimal combination of value and momentum exhausts most available gains.
  - SR of optimal combination exceeds 90% of SR of optimal strategy. 95% for horizons longer than 3.5 years.

Empirical Results

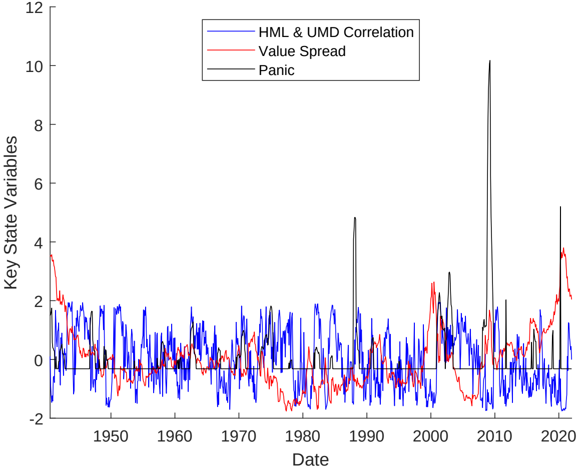
Empirical Approach

- VAR with value and momentum returns, as well as key predictors.
  - Value spread. Cohen-Polk-Vuolteenaho (2003), Campbell-Vuolteenaho (2004), Campbell-Giglio-Polk-Turley (2018)
  - Panic. (Interaction of market downturn and volatility) Daniel-Moskowitz (2016)
  - Value-momentum correlation. Our theory.

- Using VAR estimates, compute long-horizon SR and correlations.
  Campbell-Viceira (2002)

- Data and variables:
  - Monthly, 1940-2021.
  - HML and UMD factors: French's website.
  - Value spread: Constructed as in CPV, CV, CGPT.
  - Panic: Constructed as in DM.
  - Value-momentum correlation: Daily returns over the past month.

Return Predictors



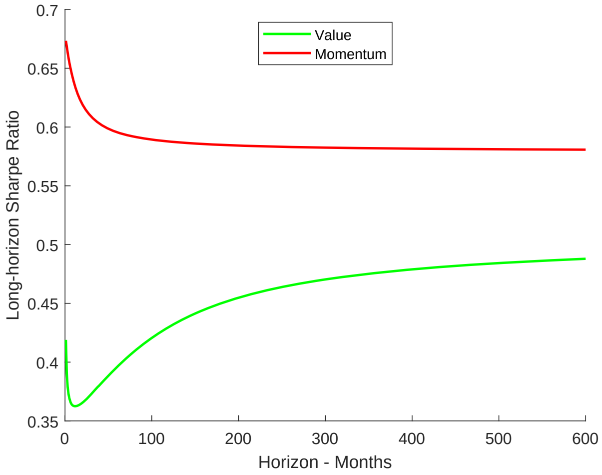
- Variables are demeaned and divided by standard deviation.
- Value-momentum correlation has mean -6% and standard deviation 52%.

VAR Coefficients

Monthly VAR: July 1940 - Dec 2021	intercept	r_{HML}	r_{MOM}	$CORR_{HML\&MOM}$	VS	PANIC	R^2
r_{HML}	**-0.01415**	**0.15334**	-0.00832	**0.00418**	**0.01166**	-11.38827	2.9%
	-2.05	**4.74**	-0.34	**2.39**	**2.58**	-1.49	
r_{MOM}	0.01604	-0.05716	-0.01689	0.00404	-0.00419	**-57.42914**	3.4%
	1.74	-1.32	-0.52	1.73	-0.70	**-5.65**	
$CORR_{HML\&MOM}$	**0.22005**	**3.48653**	-0.00340	**0.73941**	**-0.16341**	73.76268	61.3%
	2.68	**9.07**	-0.01	**35.48**	**-3.04**	0.81	
VS	**0.03865**	**-0.16348**	0.01683	-0.00248	**0.97449**	7.66643	96.7%
	4.11	**-3.72**	0.51	-1.04	**158.59**	0.74	
PANIC	**0.00001**	0.00014	**0.00008**	-0.00001	**0.00000**	**0.91011**	82.1%
	0.44	2.45	**1.85**	-1.59	**-0.30**	**66.00**	

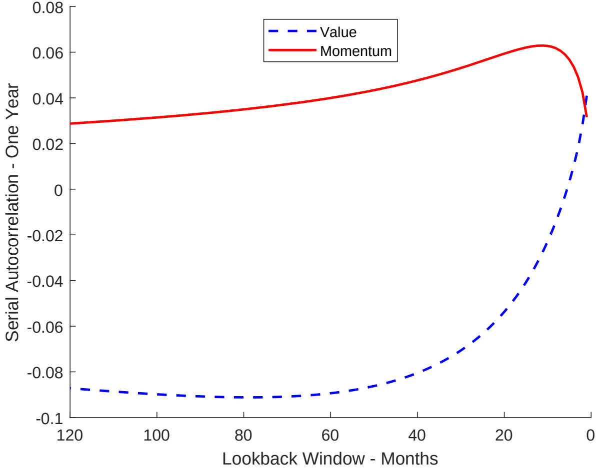
- Value returns: Predicted positively by past value returns (value has momentum), by value spread, and by value-momentum correlation.
- Momentum returns: Predicted mainly by panic (not in our theory) and positively by value-momentum correlation.

Unconditional SR



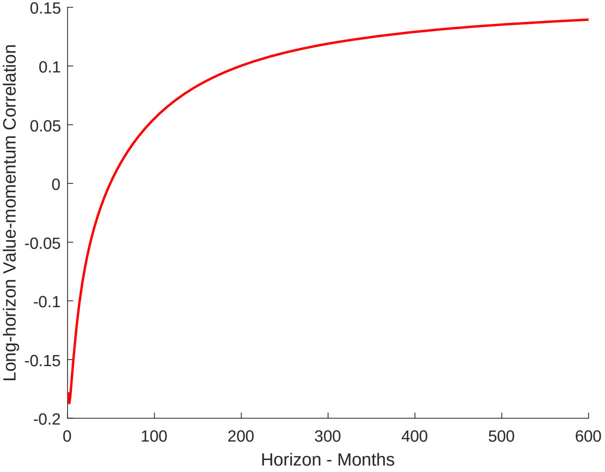
- Same dependence of SR on horizon as in our theory.
    - Effect of horizon on SR of value is weaker.
    - SR of momentum exceeds that of value for any horizon.

Autocorrelation



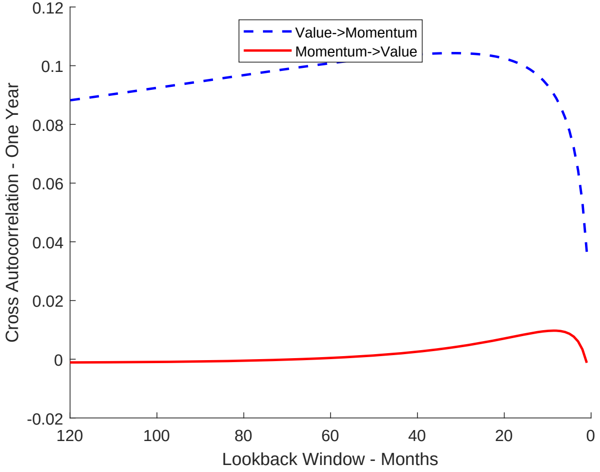
- Similar as in our theory.
  - Autocorrelation of value is not as negative.

Unconditional Value-Momentum Correlation



- Same dependence of value-momentum correlation on horizon as in our theory.

Lead-Lag Effects



- Dominant lead-lag effect is from value to momentum and is positive, as in our theory.
  - Dies out more slowly.

Conclusion

- Long-horizon investing in inefficient markets.
- Equilibrium model where momentum and value anomalies arise from capital moving slowly from underperforming to overperforming market segments.
  - Dynamic portfolio choice.
  - Empirical implications for value and momentum returns.
  - Preliminary empirical tests.
- SR, correlations, and return predictors differ significantly for short and long horizons.
  - Misleading to use short-horizon metrics to guide long-horizon investing.