What can inform us about the arrival of daily extreme stock market events?

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Presentation outline

- Motivation & contribution
- Experiment
- Results

Research questions

1. What drives unusual stock market prices distribution over time?
2. Can a measure of conditional probability of time spells of past stock market crashes predict future stock market crashes ?
3. Can we define a measure of rare disasters in stock markets that accommodates a comprehensive set of key financial and macroeconomic factors ?
4. Can such measure identify common and time-varying factors of extreme stock market movements across different industries ?

Motivation & contribution

1. Research majority examines *individual* dramatic events, e. g. a large firm, a major institution, a market.
    - Duration model: examines the *conditional probability* of dramatic events
2. Extreme stock market events as *part* of the stock market returns stochastic process.
    - Duration model: time spells reflect consecutive time units

EWS literature

1. **Agnostic** about underlying stochastic process that generates ESME

- Yoon and Park (2014): applied pattern recognition techniques to introduce stock market *instability index*
- Dopke, Fritsche, Pierdzioch (2017): boosted regression trees to re-examine usefulness of selected leading indicators in predicting recessions
- Dastkhan (2019): network representation of assets based on forward-looking conditional value-at-risk (CoVaR) model
- Chatzis, Siakoulis, Petropoulos, Stavroulakis, Vlachogiannakis (2018): extensive variations of deep learning algorithms to assess their performance in forecasting tail events in the global stock markets

EWS literature-Signals approach

1. **Specific econometric technique**

- Kaminsky (1998), Kaminsky and Reinhart (1999):
  non-parametric approach to identify thresholds as indicators of distress for certain variables
- Brüggemann and Linne (2002)
- Edison (2003)
- El-Shagi et al. (2013)

Comparison to Barro (2006)

- Attention to notion of *probabilities* of *entering* to a particular *state* of the world relative to the variable of interest (GDP sharp declines)
  - Duration model: assess factors that drive the *probability* of *entering* a particular *state* in the stock market (asset extreme returns) proxied by a time spell
- Measure of implied economic disaster probabilities renders instants of time of observed disasters consecutive
  - Duration model: time spells reflect consecutive time units
- Calibration of the disaster parameter that relates to *default* probability is *contingent* on the occurrence of a disaster
  - Duration model: *probability* of observing a market crash is *contingent* to the rate at which a time spell that initiated at a previous crash is completed (occurrence of past market crashes)

Contribution of this study

1. Develops a model for stock market crashes prediction based on duration analysis, the first empirical dynamic programming methodology.
2. Designates key time-varying financial and macroeconomic factors in extreme asset market events predictability.
3. Assesses and provides empirical evidence on significance and performance of prominent in literature financial indicators relative to extreme asset returns.
4. Suggests a risk assessment empirical tool for optimal investment allocation in the stock market.
5. Suggests a potential time-varying code of conduct for market trading rules, with the objective to eliminate the expansion of stock-market downfalls.

Key findings

1. Non-parametric estimation of extreme events differs substantially between positive and negative extreme returns.
2. Stock market crashes are significantly less likely to occur compared to extremely high stock performance (semi-parametric estimates with time-varying co-variates).
3. Differentiation in time to extreme events introduced by these indicators does not solely relate to the industry sector (distinct durations across industries).
4. Variables that capture the state of the macroeconomy strongly predict the timing of extreme asset returns.

Data

1. Monthly stock market (log-) returns of all S&P500 constituent shares.
2. January 1973-February 2019 (563 months per company, of which, 533 correspond to those once included in S&P500).
3. Financial indicators per share: price-to-earnings ratio, price-to-book ratio, dividend yield.
4. Macroeconomic variables (monthly): industrial production index growth, predominant proxy of monthly GDP growth, inflation, federal funds interest rate.

Survival function (Klein & Moeschberger, 2005)

The probability that an observation survives beyond time point $t \leq T$; the probability of entering another 'state', a 'state' of extreme stock returns

$$p(t_j) = \Pr(T = t_i), \quad i = 1, 2, 3, \ldots, \quad t_1 < t_2 < t_3 < \ldots$$

$$S(t) = Pr(T > t) = \sum_{t_i > x} p(t_i)$$

Hazard rate: *conditional* probability of extreme returns in the *next* time unit *given* no observed extreme returns up to time t.

$$h(t) = \lim_{Dt \to 0} \frac{P(t \leq T < t + Dt \mid T \geq t)}{Dt} \tag{1}$$

$$H(t) = \sum_{t_i \leq T} h(t_i) \tag{2}$$

Non-parametric estimation (K&M 2005)

Nelson-Aalen estimator:

$$\hat{S}(t) = \sum_{t_i \leq t}^{\infty} \left(1 - \frac{d_i}{y_i}\right), \quad t_i \leq t \tag{3}$$

$$\hat{S}(t) = -\exp \hat{H}(t) \tag{4}$$

If $t_1 \leq t$ does not hold, the cumulative hazard before the first observation is 0.

Are there industry (ICB) sector differences?

Hypothesis 0: $h_1(t) = h_2(t) = h_2(t) = ... = h_k(t)$ for all $t \leq T$

Hypothesis 1: At least one $h_j(t)$ differs for one or more $t \leq T$

Test function ($w_j(t_i) =$ weight per industry):

$$Z_j(t) = \sum_{i=1}^{D} W_j(t_i) \left(\frac{d_{i,j}}{Y_{i,j}} - \frac{d_i}{Y_i}\right), \quad J = 1, \ldots, K \tag{5}$$

Semi-parametric: Cox PH model expanded

Andersen & Gill (1982): Estimates of the **survival function** conditional on x_i co-variates

$$S(t \mid x_1, x_2, ...) = S_0(t) \cdot \exp(b_1 \cdot x_1 + b_2 \cdot x_2 + \dots) \tag{6}$$

$$\frac{h(t \mid x_1, x_2, ...)}{h_0(t)} = \exp(b_1 \cdot x_1 + b_2 \cdot x_2 + \dots) \tag{7}$$

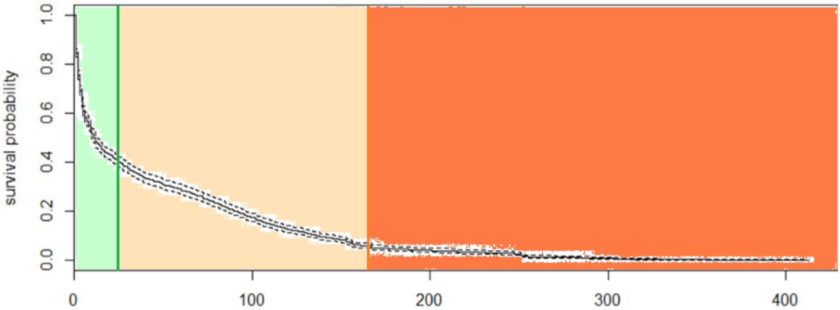
$$\ln \frac{h(t \mid x_1, x_2, ...)}{h_0(t)} = b_1 \cdot x_1 + b_2 \cdot x_2 + \dots \tag{8}$$

$h(t)$: captures *probability* that next *extreme event* will take place given not taking place up to *t*

$h(t) = \frac{f(t)}{S(t)}$; $f(t)$: probability density function of the random variable

Nelson-Allen survival curve

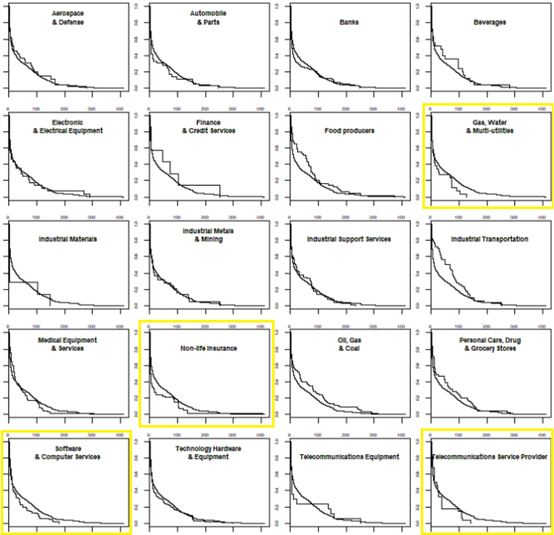
Figure 1: N-A estimate for the duration until the next extreme stock market event



Notes: This figure depicts the probabilities of the number of days that will take until the next extreme stock market event will take place as determined by the non-parametric Nelson-Allen estimates. The dotted lines indicate the 95% confidence interval.

Nelson-Allen survival curves per industry vs. market

Industries exhibiting 3σ *extreme events* more often:



Nelson-Allen survival curves per industry vs.market

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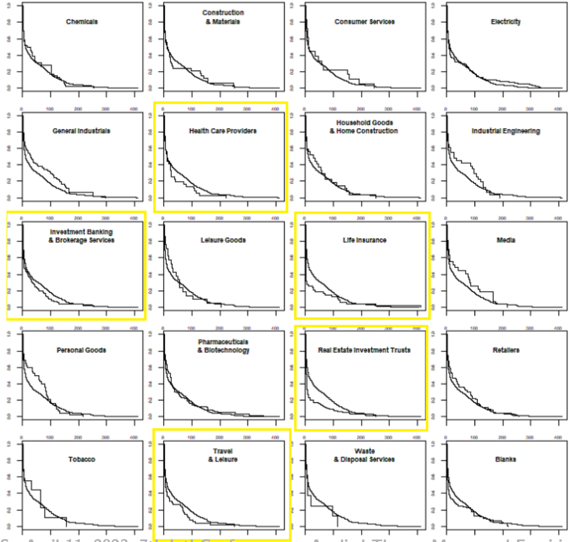
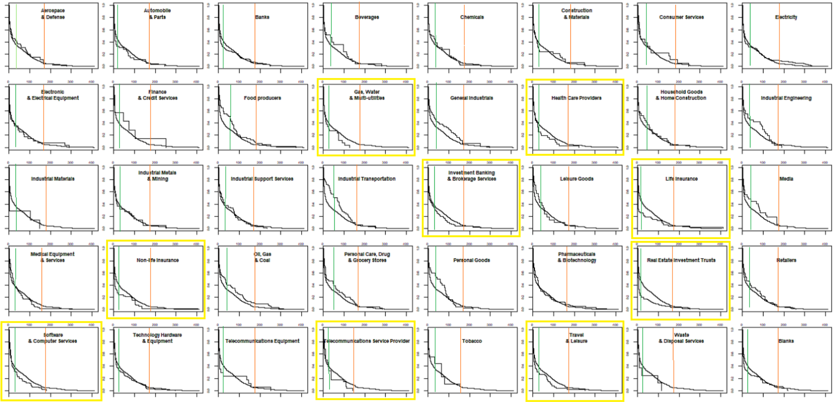
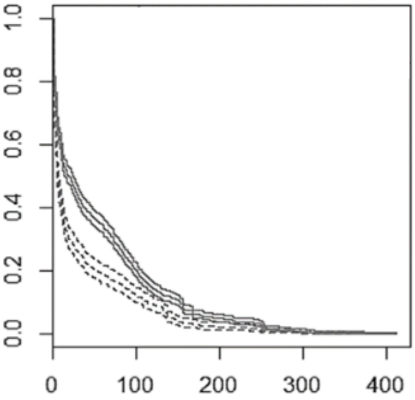


Nelson-Allen survival curves per industry vs.market

Pattern of 3 distinct ESME periods across all industries



Nelson-Allen survival curves of + and − events



Cox estimates with time-varying co-variates

$$h(t \mid Z) = h_0(t) \cdot \exp(X'_{i,t} b) \quad (9)$$

$X_{i,t}$: (1) stock returns, (2) price-to-book value, (3) div. yield, (4) price-to-earnings, (5) growth, (6) FED funds rate, (7) inflation.

Variable	Coeff.	Hazard ratios	Std.Err.(Coef.)
Stock Returns	-3.10^{***}	0.05	0.24
Price-to-book value	0.002^{*}	1.002	0.001
Dividend Yield	0.17^{**}	1.19	0.08
Price-to-earnings	-0.19^{*}	0.83	0.11
Growth	-83.32^{***}	$< 0.01\%$	6.43
Inflation	0.85^{***}	2.35	0.16
Interest Rate	-0.16^{***}	0.86	0.01

Estimation: 1137 events ($63\,475$ obs.), concordance 0.724 (Std. err. 0.009), LR test 787.9 ($p \leq 0.001$). Significance: $^{***}\ p < 0.01, ^{**}\ p < 0.05, ^{*}\ p < 0.10$.

Cox estimates with time-varying co-variates

Distinguishing between positive and negative events:

$$h(t \mid Z) = h_0(t) \cdot \exp(X'_{i,t} b) \tag{10}$$

$X_{i,t}$ contains: (1) stock returns, (2) price-to-book value, (3) div. yield, (4) price-to-earnings, (5) growth, (6) FED funds rate, (7) inflation, and **positive BS interacted with all of the above** and the constant (15 variables total)

Cox estimates with time-varying co-variates

Let $\mathbb{I}_+$ denote the indicator of a positive event.

Variable	Coeff.	Std. err.	Haz. ratio
(1) Stock Returns	-3.36^{***}	0.28	0.03
(2) Price-to-book value	0.002	0.001	1.002
(3) Dividend Yield	0.11	0.09	1.11
(4) Price-to-earnings	0.04	0.13	1.05
(5) Growth	-81.84^{***}	7.33	$< 0.01\%$
(6) Inflation	1.05^{***}	0.19	2.85
(7) Interest Rate	-0.19^{***}	0.01	0.83
(8) $\mathbb{I}_+$	1.02^{***}	0.36	2.77
(1b) Stock Returns $\cdot \mathbb{I}_+$	0.910^{**}	0.372	2.483
(2b) Price-to-book value $\cdot \mathbb{I}_+$	0.01^{***}	0	1.01
(3b) Dividend Yield $\cdot \mathbb{I}_+$	0.28	0.18	1.32
(4b) Price-to-earnings $\cdot \mathbb{I}_+$	-0.79^{***}	0.26	0.45
(5b) Growth $\cdot \mathbb{I}_+$	-0.43	13.91	0.65
(6b) Inflation $\cdot \mathbb{I}_+$	$-0.645b^{*}$	0.35	0.53
(7b) Interest Rate $\cdot \mathbb{I}_+$	0.07^{***}	0.02	1.07

1137 events ($63\,475$ obs.), concordance 0.742 (Std. err. 0.009), LR test 858.4 ($p \leq 0.001$). Significance: $^{***}\ p < 0.01, ^{**}\ p < 0.05, ^{*}\ p < 0.10$.

Conclusions & key findings I

- Strikingly similar time-to-extreme event distinct periods (durations) across industries
- Timing of extreme events significantly differs between the positive and the negative domain.
- The effects of macro-variables on 3 σ events is highly significant and persistent across Cox models.
- The rate by which output growth accumulates and reflects on asset prices is the most prominent among all variables.

Conclusions & key findings II

- Monetary policy channel: contractionary monetary policy (increase in the price of credit) decrease the probability of a 3 σ event.
- Inflation and interest rate effects differ across positive and negative 3 σ events.
- Firm's market value dominates among key firm specific factors in extreme returns' predictability.
- Investors' expectations of increased future growth rates render extreme events less frequent (price-to-earnings)
- Whilst marginally significant, increased firm's net assets (price-to-book value) render extreme events more imminent.